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On the Spectral Stefan Problem and Its Generalizations

The Stefan problem is a mathematical model for description of phase transitions from one aggregate state to another. The problem belongs to a class of nonlinear boundary value problems where the temperature of the substance and the form of the unknown dynamic interphase boundary are to be found.

In classical statement of the Stefan problem, the boundary is determined by the Stefan condition and the fact that the temperature of the substance is equal to the melting temperature (see, e.g., [1]). A more precise model for phase transitions is so-called modified Stefan problem, where the kinetic (or Hibbs-Thomson) law for the interphase boundary is used (see, e.g., [2]).

On small time-segment properties of nonlinear problem are essentially determined by properties of linearized model and in particular spectral problems. The spectral problem generated by the one-phase Stefan problem can be reduced to studying the following problem:

$$-\Delta u(x) = \lambda u(x), \qquad x \in \Omega, \tag{1}$$

$$\frac{\partial u}{\partial n}(x) = \lambda V u(x), \quad x \in \Gamma, \tag{2}$$

$$u(x) = 0, \qquad x \in S. \tag{3}$$

Here we assume that $\Omega \subset \mathbb{R}^m$ is a bounded domain with Lipshitzian boundary $\partial\Omega = \Gamma \cup S$ ($\mathrm{mes}(\Gamma \cap S) = 0$); V is a bounded positive linear operator acting in $L_2(\Gamma)$. More precise, in case of classical Stefan problem we have positive defined V, else in case of modified Stefan problem operator V is compact with polynomial asymptotic behavior of its characteristic values.

By considering the properties of the auxiliary boundary value problems we prove that eigenfunctions of problem (1)–(3) form an orthonormal basis in $H^1_{0,S}(\Omega)$; the spectrum is discrete and it consists of positive eigenvalues with polynomial asymptotic (see [3] and [4]). If dimension of Ω $m = 2$ than eigenvalues $\lambda_n = cn[1 + o(1)]$ ($n \to \infty$). If $m = 3$ than classical statement of Stefan problem implies asymptotic $\lambda_n = c_1 n^{1/2}[1 + o(1)]$ ($n \to \infty$) but modified statement implies another asymptotic $\lambda_n = c_2 n^{2/3}[1 + o(1)]$ ($n \to \infty$).

We consider also problem (1)–(3) with non-positive operator V. In this situation problem has positive and negative eigenvalues. In particular, if V is negative defined then problem has two (positive and negative) branches of eigenvalues with polynomial asymptotic and limit points $\pm\infty$.

Similar results are established for spectral problems generated by multi-component (multi-phase) Stefan problems and for generalizations on base of abstract Green's formula (see [5] and [6]).

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