A.A.Vasylenko, V.L.Maistrenko, and Yu.L.Maistrenko (Institute of Mathematics, NAS of Ukraine and National Scientific Centre for Medical and Biotechnical Research, NAS of Ukraine, Kyiv, Ukraine)

Hyperchaos in time-discrete Kuramoto model

We investigate the appearance of a high-dimensional hyperchaotic regime in a discrete analogue of the Kuramoto model [1] of globally coupled phase oscillators:

$$\psi_i^{(n+1)} = \psi_i^{(n)} + \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin\left(\psi_j^{(n)} - \psi_i^{(n)}\right), \tag{1}$$

where, $i = \overline{1, N}$, $N \in Z$ and $n = 0, 1, ..$ represents the discrete time, ψ_i are the phase variables, ω_i — the natural frequencies of the individual oscillators, and $K > 0$ is the coupling parameter. Such a discrete-time Kuramoto model has several features in common with the continuous-time system. It is well-known, however, that discrete-time models can exhibit a more complicated and richer dynamics than the respective continuous-time models [2, 3].

For the time-continuous Kuramoto model it was found that systems of at least $N \geq 4$ oscillators display a novel type of chaotic behavior called *phase chaos* [4]. For the four-dimensional time-discrete Kuramoto model we outline the region of the phase chaos in the parameter plane and determine the regions where phase chaos coexists with different periodic attractors. We also study the subcritical frequency-splitting bifurcation at the onset of desynchronization and demonstrate that the transition to phase chaos takes place via a torus destruction process [3].

For larger N hyperchaos appears in both - time-continuous and dicrete models. We use the method of Lyapunov exponent to investigate the impact of discretization on the hyperchaos, and we find that discretization of the system can decrease the number of positive Lyapunov exponents of the system.

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