Let $-\Delta, \mathcal{D}(\Delta) = W_2^2(\mathbb{R}^3)$ be the Schrödinger operator in $L_2(\mathbb{R}^3)$ and let $\mathcal{U} = \{U_t\}_{t \in (0, \infty)}$ be the collection of unitary operators $U_t f(x) = t^{3/2} f(tx)$ in $L_2(\mathbb{R}^3)$ (so-called scaling transformations).

The operator $-\Delta$ is $t^{-2}$-homogeneous with respect to $\mathcal{U}$ in the sense that

$$U_t \Delta u = t^{-2} \Delta U_t u, \quad \forall t > 0, \ u \in W_2^2(\mathbb{R}^3).$$

In other words, the set $\mathcal{U}$ determines the structure of a symmetry and the property of $-\Delta$ to be $t^{-2}$-homogeneous with respect to $\mathcal{U}$ means that $-\Delta$ possesses a symmetry with respect to $\mathcal{U}$.

Consider the heuristic expression

$$-\Delta + \sum_{i,j=1}^{\infty} b_{ij} < \psi_j, \cdot > \psi_i, \ \psi_j \in W_2^{-2}(\mathbb{R}^3), \ b_{ij} = \overline{b_{ji}} \in \mathbb{C}. \quad (1)$$

We will say that $\psi \in W_2^{-2}(\mathbb{R}^3)$ is $\xi(t)$-invariant with respect to $\mathcal{U}$ if there exists a real function $\xi(t)$ such that

$$U_t \psi = \xi(t) \psi, \quad \forall t > 0,$$

where $U_t$ is the continuation of $U_t$ onto $W_2^{-2}(\mathbb{R}^3)$.

Our aim is to study self-adjoint operator realizations of (1) assuming that all $\psi_j$ are $\xi_j(t)$-invariant with respect to the set of scaling transformations $\mathcal{U}$. In this way we generalize results of [1] to the case of infinite rank perturbations of the Schrödinger operator in $L_2(\mathbb{R}^3)$. In particular, the description of all $t^{-2}$-homogeneous extensions of the symmetric operator $-\Delta_{\text{sym}}$ is obtained. Another interesting property obtained here is the possibility to get the Friedrichs and the Krein-von Neumann extension of $-\Delta_{\text{sym}}$ as solutions of a system of equations involving the functions $t^{-2}$ and $\xi(t)$.