Svitlana Tsiupii (Kyiv, Ukraine)

Classification of three order exponent matrices

A notion of an exponent matrix is arisen from Ring theory. Some rings are given by exponent matrices, for example, tiled orders are. Exponent matrices are applied also in other mathematical theories. Each exponent matrix can be associated with some graph called a quiver and we can study such matrices by methods of Graph theory. We obtain a classification of three order exponent matrices and a list of all oriented graphs with three vertices being quivers of reduced exponent matrices [1].

We begin with some notation.

Let $M_n(Z)$ be a ring of square $n \times n$-matrices over the ring of integers.

A matrix $\mathcal{E} = (\alpha_{ij})$ from the ring $M_n(Z)$ is called *an exponent matrix* if the following conditions hold:

(i) $\alpha_{ii} = 0$ for all $i = 1, 2, \dots, n$;

(ii) $\alpha_{ik} + \alpha_{kj} \geq \alpha_{ij}$ for all $i, j, k = 1, 2, \dots, n$.

An exponent matrix $\mathcal{E} = (\alpha_{ij})$ is called *reduced* if $\alpha_{ij} + \alpha_{ji} > 0$ for all $i \neq j$.

Let $\mathcal{E}$ be a reduced exponent matrix, E be the identity one. Denote $\mathcal{E}^{(1)} = \mathcal{E} + E = (\beta_{ij})$, $\mathcal{E}^{(2)} = (\gamma_{ij})$, where

$$\gamma_{ij} = \min_k \{\beta_{ik} + \beta_{kj} - \beta_{ij}\}.$$

A graph Q is *a quiver* of an exponent matrix $\mathcal{E}$ if the adjacency matrix of Q is equal to $\mathcal{E}^{(2)} - \mathcal{E}^{(1)}$.

Theorem 1 *There are nine classes of equivalent reduced three order exponent matrices and each exponent matrix of one class has the same quiver.*

We give a description of this classes and the list of all corresponding quivers.

Theorem 2 *For each quiver with three vertices there exists a reduced exponent matrix with values of entries less than or equal to* 3.

[1] Цюпій С.І. Напівмаксимальні порядки, сагайдаки яких мають три вершини// Наукові вісті НТУУ “КПІ”. – Kyiv, 2005. – № 5 (43).