To the Inverse Problem of Stochastic Differential Systems

Let us give the system of stochastic differential equations of Ito’s type

\[
\begin{align*}
\dot{x} &= f(x, y, t), \quad x \in \mathbb{R}^n, \\
\dot{y} &= R(x, y, t) + D(x, y, t)U + \sigma(x, y, t)\xi, \quad y \in \mathbb{R}^p, \ U \in \mathbb{R}^r, \ \xi \in \mathbb{R}^k.
\end{align*}
\]

(1)

It is required to determine the control \(U\) and the diffusion matrix \(\sigma\) so that the set

\[
\Lambda(t) : \begin{cases} 
\lambda_1(x, t) = 0, \\
\lambda_2(x, y, t) = 0,
\end{cases}
\]

\[
\lambda_1 \in C_{xt}^{22}, \lambda_2 \in C_{xyt}^{121}, \lambda_1 \in \mathbb{R}^{m_1}, \lambda_2 \in \mathbb{R}^{m_2}, m_1 + m_2 = m
\]

(2)

were an integral manifold of equations’ system (1).

Here \(\{\xi_1(t, \omega), \ldots, \xi_s(t, \omega)\}\) is the system of independent Wiener processes.

The posed problem under \(\sigma \equiv 0\) is adequately investigated in [1,2] and stochastic case of reconstruction’s problem with initial stochastic differential Ito equation of second order \(\dot{x} = f(x, \dot{x}, t) + D(x, \dot{x}, t)u + \sigma(x, \dot{x}, t)\xi\) and given set \(\Lambda(t) : \lambda(x, \dot{x}, t) = 0, \ \lambda \in \mathbb{R}^m\) is considered in [3].

Under proposition \(f \in C_{xyt}^{121}\) it is proved the follow theorem by quasi-inversion method [2] with use the designations from [2,3] in combination with the Ito rule of stochastic differentiation of complicated function.

**Theorem 1.** In order that the system of equations (1) has given integral manifold (2) it is necessary and sufficient that the set of controls \(\{U\}\) and the set of coefficients’ diffusion \(\{\sigma\}\) have the form \(\{U\} = \{U_1\} \cap \{U_2\}, \{\sigma\} = \{\sigma_1\} \cap \{\sigma_2\}\), where \(U_1, U_2, \sigma_1, \sigma_2\) define in the form

\[
\begin{align*}
U_1 &= s_1[H_1C_1] + (H_1)^+(A_1 - G_1), \\
U_2 &= s_2[H_2C_2] + (H_2)^+(A_2 - G_2), \\
\sigma_1 &= s_3[H_3C_1] + (H_3)^+B_{1i}, \\
\sigma_2 &= s_4[H_4C_4] + (H_4)^+B_{2i},
\end{align*}
\]

here

\[
G_1 = \frac{\partial^2 \lambda_1}{\partial t^2} + 2 \frac{\partial^2 \lambda_1}{\partial t \partial x} f + f^T \frac{\partial^2 \lambda_1}{\partial x \partial x} f + \frac{\partial \lambda_1}{\partial x} \frac{\partial f}{\partial y} R + \frac{\partial \lambda_1}{\partial x} S_1, \quad S_1 = \frac{1}{2} \left[ \frac{\partial^2 f}{\partial y \partial y} : \sigma \sigma^T \right],
\]

\[
G_2 = \frac{\partial \lambda_2}{\partial t} + \frac{\partial \lambda_2}{\partial x} f + \frac{\partial \lambda_2}{\partial y} R + S_2, \quad S_2 = \frac{1}{2} \left[ \frac{\partial^2 \lambda_2}{\partial y \partial y} : \sigma \sigma^T \right], \quad H_1 = \lambda_{1x} f_y D, \quad H_2 = \lambda_{2y} D, \quad H_3 = \lambda_{1x} f_y, \quad H_4 = \lambda_{2y}, \quad \sigma_{1i}, \sigma_{2i}, B_{1i}, B_{1i} - i\text{-columns of matrices } \sigma_1, \sigma_2, B_1, B_2.
\]

References