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To the Inverse Problem of Stochastic Differential Systems

Let us give the system of stochastic differential equations of Ito's type

$$\begin{cases} \dot{x} = f(x, y, t), & x \in R^n, \\ \dot{y} = R(x, y, t) + D(x, y, t)U + \sigma(x, y, t)\dot{\xi}, & y \in R^p, U \in R^r, \xi \in R^k. \end{cases} \quad (1)$$

It is required to determine the control U and the diffusion matrix σ so that the set

$$\Lambda(t) : \begin{cases} \lambda_1(x, t) = 0, \\ \lambda_2(x, y, t) = 0, \end{cases} \quad \lambda_1 \in C_{xt}^{22}, \lambda_2 \in C_{xyt}^{121}, \lambda_1 \in R^{m_1}, \lambda_2 \in R^{m_2}, m_1 + m_2 = m \quad (2)$$

were an integral manifold of equations' system (1).

Here $\{\xi_1(t, \omega), \dots, \dots, \xi_k(t, \omega)\}$ is the system of independent Wiener processes.

The posed problem under $\sigma \equiv 0$ is adequately investigated in [1,2] and stochastic case of reconstruction's problem with initial stochastic differential Ito equation of second order $\ddot{x} = f(x, \dot{x}, t) + D(x, \dot{x}, t)u + \sigma(x, \dot{x}, t)\dot{\xi}$ and given set $\Lambda(t) : \lambda(x, \dot{x}, t) = 0, \lambda \in R^m$ is considered in [3].

Under proposition $f \in C_{xyt}^{121}$ it is proved the follow theorem by quasi-inversion method [2] with use the designations from [2,3] in combination with the Ito rule of stochastic differentiation of complicated function.

Theorem 1. *In order that the system of equations (1) has given integral manifold (2) it is necessary and sufficient that the set of controls $\{U\}$ and the set of coefficients' diffusion $\{\sigma\}$ have the form $\{U\} = \{U_1\} \cap \{U_2\}$, $\{\sigma\} = \{\sigma_1\} \cap \{\sigma_2\}$, where $U_1, U_2, \sigma_1, \sigma_2$ define in the form*

$$\begin{cases} U_1 = s_1[H_1C_1] + (H_1)^+(A_1 - G_1), \\ U_2 = s_2[H_2C_2] + (H_2)^+(A_2 - G_2), \end{cases} \quad \begin{cases} \sigma_{1i} = s_3[H_3C_1] + (H_3)^+B_{1i}, \\ \sigma_{2i} = s_4[H_4C_4] + (H_4)^+B_{2i}, \end{cases} \quad \text{here}$$

$$G_1 = \frac{\partial^2 \lambda_1}{\partial t^2} + 2 \frac{\partial^2 \lambda_1}{\partial t \partial x} + \frac{\partial^2 \lambda_1}{\partial x \partial x} f + f^T \frac{\partial^2 \lambda_1}{\partial x \partial x} f + \frac{\partial \lambda_1}{\partial x} \frac{\partial f}{\partial y} R + \frac{\partial \lambda_1}{\partial x} S_1, \quad S_1 = \frac{1}{2} \left[\frac{\partial^2 f}{\partial y \partial y} : \sigma \sigma^T \right],$$

$$G_2 = \frac{\partial \lambda_2}{\partial t} + \frac{\partial \lambda_2}{\partial x} f + \frac{\partial \lambda_2}{\partial y} R + S_2, \quad S_2 = \frac{1}{2} \left[\frac{\partial^2 \lambda_2}{\partial y \partial y} : \sigma \sigma^T \right], \quad H_1 = \lambda_{1x} f_y D, \quad H_2 = \lambda_{2y} D,$$

$$H_3 = \lambda_{1x} f_y, \quad H_4 = \lambda_{2y}; \quad \sigma_{1i}, \sigma_{2i}, B_{1i}, B_{2i} - i\text{- columns of matrices } \sigma_1, \sigma_2, B_1, B_2.$$

References

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 3. Tleubergenov M.I. //Differentsial'nye uravneniya. — Moscow, 5(2001), 714-716.
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