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A criterion of lineal convexity of complete Hartogs domain in $\mathbb{H}^2$

Notion of lineal convexity is one of the most impotent in complex analysis. In the work we considered analogs of this notion and some related to it theorems in the case of n-dimensional quaternion space $\mathbb{H}^n$. A domain $D \subset K^n$, $n \geq 2$ is called *locally linearly convex* if through every boundary point there passes $(n-1)$-dimensional hyperplane Π_K which does not intersect domain D. (K means field of complex numbers $\mathbb{C}$ or algebra of quaternions $\mathbb{H}$; multiplication by scalar me make from the left side, in consequence of noncommutativity of quaternions).

Theorem. *Let*

$$\Omega = \{(z,t) \in \mathbb{H}^2 : \ |t| < R(z)\}$$

be bounded complete Hartogs domain in $\mathbb{H}^2$ with boundary of class C^2 and $R_z \neq 0$ everywhere on the boundary. Domain Ω is lineally convex if and only if at every boundary point $w = (z_0, t_0)$

$$\omega(w,s) \geq 0$$

where

$$\omega(w,s) = \sum_{i,j=1}^{n} \sum_{k,l=0}^{3} \frac{\partial^2 \rho(w)}{\partial z_i^k \partial z_j^l} s_j^l s_i^k = \sum_{i,j=1}^{n} \sum_{k,l=0}^{3} s_j^l \frac{\partial^2 \rho(w)}{\partial z_i^k \partial z_j^l} s_i^k = \sum_{i,j=1}^{n} \sum_{k,l=0}^{3} s_j^l s_i^k \frac{\partial^2 \rho(w)}{\partial z_i^k \partial z_j^l}$$

for all nonzero vectors $s = (s_1, s_2, \ldots, s_n) \in \mathbb{H}^n$, $\|s\| = 1$ satisfying the equation

$$\sum_{i=1}^{n} \frac{\partial \rho(w)}{\partial z_i} s_i = 0.$$