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Degenerate Bifurcations and Border Collision in One- and Two-dimensional Piecewise Smooth Maps

Bifurcations phenomena occurring in non-smooth discrete dynamical systems have got an increasing research interest nowadays mainly due to numerous applied models, defined by continuous or discontinuous piecewise smooth functions (see, e.g., [1], [3]). While the bifurcation theory for smooth systems is quite well developed, for the piecewise smooth systems it is still far from a complete form. For such systems, besides standard smooth bifurcations, new dynamic phenomena can be observed, related to the existence of borders separating the regions of different definition of the map. An invariant set in the phase space of the piecewise smooth map may collide with such a border, and this collision may lead to a bifurcation often followed by drastic changes, such as, for example, transition from an attracting fixed point to a chaotic attractor. Bifurcation phenomena occurring due to the border collision are now collected under the term *Border Collision Bifurcations*, first introduced by Nusse and Yorke in 1992 [2].

The purpose of our work is to consider particular bifurcations observed in piecewise smooth maps, correlated with the border collision bifurcations, which we call *degenerate bifurcations*, occurring when an eigenvalue at the fixed point of the map (locally smooth) crosses the unit circle in presence of some degeneracy, when the standard bifurcation theorems known for smooth maps are not applied. We first recall these theorems considering the degeneracy inherent in linear and linear-fractional maps. This will lead to the definition of degenerate bifurcations for generic piecewise smooth maps. Then we present several examples of the degenerate bifurcations.

- [1] Di Bernardo M., Budd C.J., Champneys A.R., Kowalczyk P. Piecewise-Smooth Dynamical Systems. Theory and Applications. Springer-Verlag, London, 2008.
 - [2] Nusse H.E., Yorke J.A. “Border-collision bifurcations including period two to period three for piecewise smooth systems”, *Physica D* 57, 39–57, 1992.
 - [3] Z.T. Zhusubaliyev Z.T., Mosekilde E. Bifurcations and Chaos in Piecewise-Smooth Dynamical Systems. World Scientific, Singapore, 2003.
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