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Support propagation properties of solutions for some quasilinear parabolic equations

We study the support propagation properties of energy solutions to the Cauchy problem for a wide class of quasilinear parabolic equations of the following type:

$$\begin{cases} u_t - \triangle u + h(t)|u|^{q-1}u = 0 & \text{in } \Omega \times (0,T), \\ u(x,0) = u_0(x) & \text{on } \Omega, \end{cases}$$

where $\Omega \subset R^N$, $N \geq 1$, is a bounded domain with C^1–boundary, $0 < q < 1$, the initial function $u_0 \in L_2(\Omega)$, $R^N \setminus \{\operatorname{supp} u_0\} \neq \emptyset$. Here $h(t)$ is a continuous, nonnegative, nondecreasing function, such that $h(0) = 0$. It is well known that if $h(t) \geq h_0 > 0$ for every $t > 0$, then a solution of these problem has the finite speed propagation property.

We obtain the energy estimates of Saint-Venant's type for the energy solutions for the problem mentioned above. We are looking for the dependence between the support propagation properties of solutions and a degeneration degree of the absorption $h(t)$–potential.

Our main result is the sharp sufficient conditions on the degeneracy of the absorption potential which guarantee the finite speed propagation of supports of the solutions.

The proof is based on the local energy method in the spirit of paper [1].

[1] Belaud Y., Shishkov A.E. Long-time extinction of solutions of some semilinear parabolic equations// Journal of Differential Equations — 2007. — **238**. — P. 64–86.