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Some peculiarities of stability of stochastic difference equations with continuous time

The problems of stability are considered here for stochastic difference equations with continuous time, which are popular enough in researches (see [1]-[3] and references therein).

Consider the difference equation with continuous time

$$x(t+1) = ax(t) + bx(t-1) + \sigma x(t)\xi(t+1), \qquad t > -1, \tag{1}$$

$$x(s) = \phi(s), \qquad s \in [-2, 0],$$

Here a, b and σ are constants, a stochastic process $\xi(t)$ satisfy the conditions $\mathbf{E}\xi(t) = 0$, $\mathbf{E}\xi^2(t) = 1$, $t > 0$.

Definition 1.1. The trivial solution of equation (1) is called mean square stable if for any $\epsilon > 0$ there exists a $\delta > 0$ such that $\mathbf{E}|x(t;\phi)|^2 < \epsilon$ for all $t \geq 0$ if $\|\phi\|^2 = \sup_{\theta\in\Theta} \mathbf{E}|\phi(\theta)|^2 < \delta$.

Definition 1.2. The trivial solution of equation (1) is called asymptotically mean square stable if it is mean square stable and for each initial function ϕ $\lim_{t\to\infty} \mathbf{E}|x(t;\phi)|^2 = 0$.

Definition 1.3. The trivial solution of equation (1) is called asymptotically mean square quasistable if it is mean square stable and for each $t \in [0, 1)$ and each initial function ϕ $\lim_{j\to\infty} \mathbf{E}|x(t+j;\phi)|^2 = 0$.

It is evidently that asymptotic mean square quasistability follows from asymptotic mean square stability but the inverse statement is not true.

It is known [1] that the necessary and sufficient condition for asymptotic mean square quasistability of the trivial solution of equation (1) is

$$|b| < 1, \qquad |a| < 1 - b, \qquad \sigma^2 < [(1-b)^2 - a^2]\frac{1+b}{1-b}.$$

The problem is considered to get similar stability and quasistability conditions for integro-difference equation

$$x(t+1) = ax(t) + b\int_{t-1}^{t} x(s)ds + \sigma x(t)\xi(t+1), \qquad t > -1.$$

[1] Shaikhet L. Construction of Lyapunov functionals for stochastic difference equations with continuous time, Math. Comput. Simulation 66 (6) (2004) 509-521.

[2] Shaikhet L. General method of Lyapunov functionals construction in stability investigations of nonlinear stochastic difference equations with continuous time. Special Issue "Stochastic Dynamics with Delay and Memory", Stochastics and Dynamics 5 (2) (2005) 175–188.

[3] Shaikhet L. Some new aspect of Lyapunov type theorems for stochastic difference equations with continuous time, Asian Journal of Control 8 (1) (2006) 76–81.