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Justification of *c*-Number Substitution in Bogolyubov's Superfluidity Theory Hamiltonian

In 1947 Bogolyubov introdused the model Hamiltonian of the superfluidity theory [1, 2],

$$\hat{H}^{\mathrm{B}}_{\Lambda} = \sum_{k\in\Lambda^*} \epsilon_k \hat{a}^\dagger_k \hat{a}_k + \frac{1}{2V}\sum_{k\neq 0} \nu(k)(\hat{a}^\dagger_k \hat{a}^\dagger_{-k}\hat{a}^2_0 + \text{c.c.}) + \frac{\hat{a}^\dagger_0\hat{a}_0}{V}\sum_{k\neq 0}[\nu(k)+\nu(0)]\hat{a}^\dagger_k\hat{a}_k + \frac{\nu(0)}{2V}\hat{a}^{\dagger 2}_0\hat{a}^2_0.$$

Here $\hat{a}^\dagger_k, \hat{a}_k$ are the usual boson creation (annihilation) operators, $\epsilon_k = |k|^2/(2m)$ is the one-particle energy spectrum of free bosons in the modes $k \in \Lambda^*$ (we propose $\hbar = 1$), $\nu(k)$ is the Fourier transform of the interaction pair potential $\Phi(x)$. We suppose that $\Phi(x) = \Phi(|x|) \in L^1(\mathbb{R}^3)$ and $\nu(k)$ is a real function with a compact support such that $0 \leq \nu(k) = \nu(-k) \leq \nu(0)$ for all $k \in \mathbb{R}^3$. Bogolyubov proposed to use a c-number substitution for the most relevant operators in this Hamiltonian, namely $\hat{a}^\dagger_0/\sqrt{V} \to \bar{c}$, $\hat{a}_0/\sqrt{V} \to c$, where $c \in \mathbb{C}$.

The rigorous justification for this substitution in the case of total, correct pair Hamiltonian was done in a classic paper of Ginibre [3] in 1968. In 2005 Lieb, Seiringer and Yngvason [4] developed the Ginibre's method with a fair degree of generality by the Berezin–Lieb inequality.

Here the validity of substituting a c-number for the $k = 0$ mode operators $\hat{a}^{\#}_p$ is established rigorously for the Bogolyubov's Hamiltonian, thereby verifying one aspect of Bogolyubov's 1947 theory. Our analysis confirms an assertion, that if the system is stable after the c-number substitution, then so is the original one (in contrast to the papers [5, 6]). We have shown that if the potential $\nu(k)$ in the Bogolyubov model satisfies the condition $2\nu(0) \geq (2\pi)^{-3}\int_{\mathbb{R}^3} d^3k \epsilon_k^{-1}\nu^2(k)$, then there exists the domain of stability on the phase diagram $\{0 < \mu \leq \mu^*, 0 \leq \theta \leq \theta_0(\mu)\}$, where the nontrivial solution of the self-consistency equation takes place (μ is the chemical potential, θ is the temperature). In this domain there is the non-zero Bose condensate. At the boundary $\theta = \theta_0(\mu)$ of this domain the Bose condensate density equals $\rho_0 = \mu/\nu(0)$. In this case the quasi-particles spectrum of the Bogolyubov Hamiltonian $E_k = \sqrt{\epsilon_k(\epsilon_k + 2\rho_0\nu(k))}$ has a gapless type and the famous criterion of superfluidity $\min_k(E_k/|k|) > 0$ holds.

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