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Influence of Viscosity on Oscillating Transition in Hydrodynamics

Turbulent regimes are usually arisen under a small viscosity (or equivalently under a high Reynolds number) and are sometimes associated with rapidly oscillating fluid dynamics. Moreover, in numerical modeling it is known that rapidly oscillation effects arise under computer simulations of solutions of Navier–Stokes equations with a vanishing viscosity. But reasons of the effects are not clear, since the effects may be turbulent regimes or the numerical simulations may be incorrect. Some theoretical results in the direction will be presented here. The results are based on homogenization theory and are full for nonstationary linearized equations of hydrodynamics with periodic rapidly oscillating data and a vanishing viscosity. Some of the results hold for Navier–Stokes equations.

Homogenization of nonstationary linearized equations of hydrodynamics and Navier–Stokes equations with periodic rapidly oscillating initial data and the vanishing viscosity will be discussed. We give homogenized (limit) equations whose solutions determine approximations (leading terms of the asymptotics) of the solutions of the equations under consideration and estimate the accuracy of the approximations. These approximations and estimates shed light on the following interesting property of the solutions of the equations. When the viscosity is not too small, the approximations contain no rapidly oscillating terms, and the equations under consideration asymptotically smooth the rapid oscillations of the data; thus, the equations are asymptotically parabolic. If the viscosity is very small, the approximations can contain rapidly oscillating terms with zero means, and the equations are asymptotically hyperbolic.

Thus, the equations with a very small viscosity have the property of space-time resonance with respect to spatial oscillations of the data. On the other hand, the equations have smoothing property when the viscosity is not too small. The methods used also apply to the nonstationary Navier–Stokes equations if the rapid oscillations of data are zero in mean and the viscosity is not too small. The smoothing property holds also in the case. Moreover the solutions are asymptotically small and it is valid for arbitrarily small viscosity in the case of potential data.

The homogenization of some cases of nonstationary linearized equations of hydrodynamics and Navier–Stokes equations with periodic rapidly oscillating "forces" were considered in [1]–[3]. In particular, the results are applicable to some Kolmogorov flows.

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[2] Sandrakov G. V. // Doklady Math. — 2007. — **75**, N 2.

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