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\textbf{\textit{p}}-adic operational calculus of Mikusinski type

We consider Operational Calculus of Mikusinski type \cite{1,2} based on space \(C(Z_p)\) of continuous functions from the ring of \(p\)-adic integers \(Z_p\) to the field of \(p\)-adic numbers \(Q_p\) \cite{3,4}. We use canonical embedding and corresponding convolution

\[ \mathcal{F} : C(Z_p) \ni f \rightarrow \hat{f}(y) := \sum_{n=0}^{+\infty} f(n)y^n \in Q_p[[y]], \quad f \ast g := \mathcal{F}^{-1}[(\mathcal{F}f) \cdot (\mathcal{F}g)]. \]

Space \(C(Z_p)\) with pointwise addition and convolution multiplication becomes an algebra. Standard norm

\[ \|f\| = \sup_{x \in Z_p} |f(x)|_p \]

turns out to be a valuation, and \(C(Z_p)\) is complete with respect to it.

We introduce a fraction field \(C/C\) and call it \(p\)-adic Mikusinski field or field of \(p\)-adic hyperfunctions. Surprisingly, \(C/C\) is incomplete with respect to valuation continued from \(C(Z_p)\).

We choose certain hyperfunctions corresponding to operations of shift \(L \sim 1/y\), difference operator \(\Delta \sim 1/y - 1\), operator of indefinite summation \(S = \Delta^{-1}\), differentiation and integration. \(p\)-Adic exponential function \(a^x \in C(Z_p)\), \(a \in 1+pZ_p\), is a particular case of \(p\)-adic hyperfunction \(E_a := L/(L - a)\), \(a \in Q_p\).

Mahler base in \(C(Z_p)\) is easily described using \(p\)-adic hyperfunctions:

\[ \left\{ \binom{x}{n} \right\} = \frac{x(x - 1)...(x - n + 1)}{n!} = S^n(1 + S) \in C/C, \quad n = 0, 1, 2, \ldots . \]

Using Mahler expansion we embed convolution algebra of continuous functions \(C(Z_p)\) and convolution algebra of \(p\)-adic measures \(C(Z_p)'\) into one field with valuation. More singular distributions are described in the same way.