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Independent sets and partitions of infinite graphs

Let $\Gamma(V,E)$ be a connected graph with the set of vertices V and the set of edges E, d be the path metric on V, $B(v,r)=\{x\in V : d(v,x)\le r\}$ be the ball of radius r with the center v. A subset $A\subseteq V$ is of finite index if there exists m such that $B(A,m)=V$. In this case, we define the index of A by $ind\ A = min\{m : B(A,m)=V\}$.

Let $\Gamma(V,E)$ be a graph, r be a natural number such that $|V|\ge r$. By [1, Theorem 1], there exists a partition $V=V_1\bigcup V_2\bigcup\ldots\bigcup V_r$ such that $ind\, V_i\le r-1$ for each $i\in\{1,\ldots,r\}$.

Theorem 1. *For every infinite graph $\Gamma(V,E)$, there exists a partition $V=V_1\bigcup V_2\bigcup\ldots$ such that each subset V_i is of finite index.*

Theorem 2. *Let $\Gamma(V,E)$ be a graph, m be a natural number, κ be a cardinal. If $\mid B(v,m)\mid\ge\kappa$ for every $v\in V$, then V can be partitioned in κ subsets of index $\le 3m$.*

Theorem 3. *Let $\Gamma(V,E)$ be an infinite graph, κ be a limit cardinal, $|V|=\kappa$. Suppose that, for each $\kappa'<\kappa$, there exists a natural number m such that $\mid B(v,m)\mid\ge\kappa'$ for every $v\in V$. Then V can be partitioned in κ subsets of finite index.*

Theorem 4. *Let $\Gamma(V,E)$ be an infinite graph, κ be an infinite cardinal such that $\mid B(v,1)\mid\ge\kappa$ for each $v\in V$. Then V can be partitioned in k subsets of index* 1.

For proofs of above theorems, we used the independent subsets of graphs and some its generalizations.

[1] K.D. Protasova, *Balanced partition of graphs*, Matem. Zamet., **79** (2006), 127-132.