Estimation in an implicit linear multivariate measurement error model

Consider the model of observations $DZ \approx 0$, where $Z \in \mathbb{R}^{(n+p)\times p}$ is an unknown matrix parameter, which has to be estimated and the data matrix $D \in \mathbb{R}^{m\times(n+p)}$ is observed with errors. The main assumption was that we observe two independent copies of the model: $D(k)Z \approx 0$, $k = 1, 2$, where $D(k) \in \mathbb{R}^{m\times(n+p)}$ are observed, $D(k) = \bar{D}(k) + \tilde{D}(k)$, $\bar{D}(k)Z = 0$, $k = 1, 2$. Here $\bar{D}(k)$ are unknown nonrandom matrices and $\tilde{D}(k)$ are error matrices. We assumed that $\dim \ker \bar{D}(k) = p$, $k = 1, 2$, and total error covariance structure of data matrix is known up to two scalar factors. There exists $n_1$, $1 \leq n_1 \leq n + p - 1$, such that $\tilde{D}(k) = \begin{bmatrix} \tilde{D}_1(k) & \tilde{D}_2(k) \end{bmatrix}$, $\tilde{D}_1(k) \in \mathbb{R}^{m_1\times n_1}$ and $E \tilde{D}_1^T(k)\tilde{D}_2(k) = 0$, and $E \tilde{D}_j^T(k)\tilde{D}_j(k) = \lambda^0_j W_j(k)$, $j = 1, 2$, where $W_j(k)$ are known positive semidefinite matrices and $\lambda^0_j$ are unknown positive scalars. The estimator of $V_p := \ker \bar{D}(1) = \ker \bar{D}(2)$ for increasing $m_1$ and $m_2$ under fixed $n$ and $p$ is constructed.

Based on the method of corrected objective function, the estimators $\hat{\lambda} := (\hat{\lambda}_1, \hat{\lambda}_2)$ of the scalar factors are proposed. Sufficient conditions of the consistency of the estimators are given.

Using both clusters we constructed a single subspace: $D_c := \begin{bmatrix} D(1) \\ D(2) \end{bmatrix}$ and $W_{c_j} := W_j(1) + W_j(2)$, $j = 1, 2$. Then $D_c = \bar{D}_c + \tilde{D}_c$ and $H := E \tilde{D}_c^T\tilde{D}_c = \begin{bmatrix} \lambda^0_1 W_{c_1} & 0 \\ 0 & \lambda^0_2 W_{c_2} \end{bmatrix}$. Define the matrix $\hat{H} = D_c^T D_c - \begin{bmatrix} \hat{\lambda}_1 W_{c_1} & 0 \\ 0 & \hat{\lambda}_2 W_{c_2} \end{bmatrix}$. Let $V_p(\hat{H})$ denotes the subspace spanned by the first $p$ eigenvectors of $\hat{H}$ corresponding to the smallest eigenvalues, then $\hat{V}_p = V_p(\hat{H}) = \text{span}(\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_p)$. Sufficient conditions of the consistency of the estimator $\hat{V}_p$ are given.

