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On local deformations of quadratic forms

By a quadratic form we mean here a quadratic form over the field $\mathbb{R}$

$$f(z) = f(z_1, \dots, z_n) = \sum_{i=1}^{n} f_i z_i^2 + \sum_{i<j} f_{ij} z_i z_j.$$

The set of all such form with $f_1, \dots, f_n = 1$ is denoted by $\mathcal{R}_0$.

Let $f(z) \in \mathcal{R}_0$ and $s \in \{1, \dots, n\}$. We introduce the notion of *the s-deformation of* $f(z)$ as follows:

$$f^{(s)}(z, a) = f^{(s)}(z_1, \dots, z_n, a) = a z_s^2 + \sum_{i \neq s} z_i^2 + \sum_{i<j} f_{ij} z_i z_j,$$

where a is a parameter. Denote by $F_+^{(s)}$ the set of all $b \in \mathbb{R}$ such that the form $f^{(s)}(z, b)$ is positive definite, and put $F_-^{(s)} = \mathbb{R} \setminus F_+^{(s)}$. In other words, $b \in F_-^{(s)}$ iff there exists a nonzero vector $r = (r_1, \dots, r_n) \in \mathbb{R}^n$ such that $f^{(s)}(r_1, \dots, r_n, b) \leq 0$. Further, put

$$m_f^{(s)} = \sup \mathrm{F}_-^{(\mathrm{s})} \in \mathbb{R} \cup \infty$$

(since $x \in F_-^{(s)}$ implies $y \in F_-^{(s)}$ for any $y < x$, this supremum is a limit point). We call $m_f^{(s)}$ *the s-th P-number of* $f(z)$. It is easy to see that if $f(z_1, \dots, z_n) \in \mathcal{R}_0$, then $m_f^{(s)} \geq 0$.

Theorem *Let $f(z_1, \dots, z_n) \in \mathcal{R}_0$ and let $m_f^{(s)} \neq \infty$. Then*

1) $m_f^{(s)} \in F_-^{(s)}$, and consequently $m_f^{(s)}$ (is the greatest number of $F_-^{(s)}$).

2) the form $f^{(s)}(z, m_f^{(s)})$ is non-negative definite.