Volodymyr Lagoda, Igor Parasyuk (National Taras Shevchenko University of Kyiv, Ukraine)

On existence of global V-bounded solutions for nonlinear systems of ODEs

The goal of this report is to lay down sufficient conditions under which the nonlinear nonautonomous system of ODEs

\[ \dot{x} = f(t, x) \quad (1) \]

where \( f : \Omega \mapsto \mathbb{R}^n \) (\( \Omega \subset \mathbb{R}^{1+n} \)) has a global solution \( x(t) \) extendable on the entire time axis and possessing the property that a given auxiliary function \( V(t, x) \) (a time dependent norm surrogate) is bounded along the graph of \( x(t) \). We especially focus on getting estimates for the function \( V(t, x(t)) \). The main results are obtained by using the Ważewski topological principle.

To apply the Ważewski principle, along with the function \( V \) we use another auxiliary function \( W(t, x) \). In general case, this function is a sign-changing one, but it must have positive derivative by virtue of the system (1) in the domain where \( V \geq v_0 \) for some constant \( v_0 > 0 \). We call \( V \) and \( W \) the estimating function and the guiding function respectively and we say that together they form the V–W-pair of the system.

We prove two main theorems concerning the existence and the uniqueness of V-bounded solution to a nonlinear nonautonomous system possessing V–W-pair. We also show how the estimating and guiding functions are constructed by means of nonautonomous quadratic forms.