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Approximation of Continuous Periodic Functions by Linear Methods

Let C be the space of 2π-periodic continuous functions f with the norm $\|f\|_C = \max\limits_t |f(t)|$. Let $C^\psi_\beta H_\omega$ be the set of functions $f \in C$ representable in the form of a convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int_{-\pi}^{\pi} \varphi(x-t)\Psi_\beta(t)\,dt, \quad \varphi \in H^0_\omega,$$

where $H^0_\omega = \{\varphi \in C : \ |\varphi(t') - \varphi(t'')| \le \omega(|t'-t''|) \ \forall\, t', t'' \in R, \ \varphi \perp 1\}$, $\omega(t)$ is an arbitrary modulus of continuity, $\Psi_\beta(t)$ is a summable function, which the Fourier series is of the form $\sum_{k=1}^\infty \psi(k)\cos(kt - \beta\pi/2)$, $\beta \in R$, $\psi(k)$ is prescribed number of sequences. Let M'_0 be the set of continuous functions $\psi(t)$ convex below for all $t \ge 1$, having at the points $t = k$ the values $\psi(k)$ and which satisfy conditions: $\lim\limits_{t\to\infty} \psi(t) = 0$, $0 < \frac{t}{\psi^{-1}(\frac{\psi(t)}{2}) - t} \le K$, $t \ge 1$ and $\int\limits_1^\infty \frac{\psi(t)}{t}\,dt < \infty$.

Let

$$Z^{\psi,A}_n(f,x) = \frac{a_0}{2} + \sum_{k=1}^{n-1}\left(1 - \frac{\psi(n)}{\psi(k)}\Big(A + (1-A)\frac{k}{n}\Big)\right)(a_k \cos kx + b_k \sin kx),$$

where $n \in N$, $\psi \in M'_0$, $A > 0$ and a_k, b_k is the Fourier coefficients of function $f(x)$.

For the class $C^\psi_\beta H_\omega$, where $\psi \in M'_0$, it is proved the following statement.

Theorem. *Let $\psi \in M'_0$, $\omega(t)$ is an arbitrary modulus of continuity, $\beta \in R$ and $A > 0$. Then the following relation holds as $n \to \infty$*

$$\sup_{f \in C^\psi_\beta H_\omega} \|f(x) - Z^{\psi,A}_n(f,x)\|_C = \frac{\theta_n(\omega)}{\pi}\Big|\sin\frac{\beta\pi}{2}\Big|\left(A\psi(n)\int_{1/n}^1 \frac{\omega(2t)}{t}\,dt + \int_0^{1/n} \psi(\frac{1}{t})\frac{\omega(t)}{t}\,dt\right) +$$

$$+ O(1)\psi(n), \tag{1}$$

where $\theta_n(\omega) \in [\frac{2}{3}, 1]$, $\theta_n(\omega) = 1$ if $\omega(t)$ is a convex modulus of continuity, and $O(1)$ is a quantity uniformly bounded in n and β.

For the class $W^r_\beta H_\omega$ and Zygmund method Z^s_n the equality (1) is proved in work [1]. For a case $A = 1$ the equality (1) is obtained in [2].

[1] Efimov A.V. Linear methods of approximating certain classes of continuous periodic functions (Russian) // Tr. Mat. Inst. Akad. Nauk SSSR. — 1961. — **62**. — P. 3—47.

[2] Serdyuk A.S., Ovsiy E.Y. Approximation of classes $C^\psi_\beta H_\omega$ by generalized sums of Zygmund (Russian) // Ukrain. Mat. Zh. — 2009. — **61**, N 4. — P. 524—537.