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On the iterative solution of the Wiener-Hopf equation with a probabilistic kernel

Consider Wiener-Hopf equation with an unknown function $\varphi(\cdot)$ of the form

$$\varphi(x) = -\int_0^{+\infty} \varphi(y) dK(x-y) + f(x), \qquad 0 \leq x < +\infty, \tag{1}$$

where $K(\cdot)$ is a monotonic function and the integral is the Lebesgue-Stieltjes one. We are interested in a solution $\varphi(\cdot)$ of the equation such that

$$\varphi(x) \leq c < +\infty, \qquad \lim_{x\to+\infty} \varphi(x) = c, \tag{2}$$

where c is a known limit. In the report we present the necessary and sufficient conditions of existence and sufficient conditions for the uniqueness of the solution of problem (1), (2) and substantiate an iterative method for its solution [1]. In contrast to papers [2, 3], which analyze the Wiener-Hopf equation by probabilistic methods, we use here the general ideas of functional analysis.

Assumptions

1. a) $K(\cdot)$ is a continuous distribution function; b) $K(x) > 0$ or $K(x) < 1$ for all x.

2. $f(x)$ is bounded, continuous, and satisfies the condition $f(x) \leq c\,(1 - K(x))$ on $[0, +\infty)$.

3. There exists an integrable function $\varphi^*(x)$ such that

$$a)\ \ \varphi^*(x) \leq c; \qquad b)\ \ \lim_{x\to+\infty} \varphi^*(x) = c; \qquad c)\ \ \varphi^*(x) + \int_0^{+\infty} \varphi^*(x) dK(x-y) \leq f(x).$$

Theorem 1. If assumptions 1a, 2 hold, then condition 3 is necessary and sufficient for the existence, and assumptions 1-3 are sufficient for the existence and uniqueness of the solution of (1), (2).

Theorem 2. In assumptions 1-3 the sequence of approximations

$$\varphi^{k+1}(x) = -\int_0^{+\infty} \varphi^k(y) dK(x-y) + f(x), \qquad \varphi^* \leq \varphi^0 \leq c, \qquad k = 0, 1, ...,$$

starting from some initial function $\varphi^0(x)$, such that $\varphi^*(x) \leq \varphi^0(x) \leq c$ for all $x \in [0, +\infty)$, point-wise converges to the solution of the problem (1), (2).

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[2] Spitzer F. The Wiener-Hopf equation whose kernel is a probability density // Duke Math. J. — 1957. — **24**, N 3. — P.327-344.

[3] Asmussen S. A probabilistic look at the Wiener-Hopf equation // SIAM Review. — 1998. — **40**, N 2. — P.198-201.