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## Some Goodness-of-Fit Tests Based on Estimates of Kernel Type Distribution

Let $X_1, X_2, \dots, X_n$ be a sequence of independent equally distributed random values having a distribution density $f(x)$. Using the sampling $X_1, X_2, \dots, X_n$, it is required to check the hypothesis $H_0 : \ f(x) = f_0(x)$. Here we consider the test of the hypothesis $H_0$ based on the statistic $U_n = na_n^{-1} \int (f_n(x) - f_0(x))^2 r(x)\, dx$, where $f_n(x)$ is the recurrent Wolverton–Wagner kernel estimate of the probability density defined by $f_n(x) = n^{-1} \sum\limits_{i=1}^{n} a_i K(a_i(x - X_i))$, where $\{a_i\}$ is an increasing sequence of positive integers tending to infinity, $r(x) \in R$ ($R$ is the set of non-negative, bounded and piecewise-continuous functions at $(-\infty, +\infty)$), $K(x) \in H = \{h : \ h(x) \geq 0, \ \sup\limits_{x \in (-\infty, +\infty)} h(x) < \infty, \ \int h(x)\, dx = 1, \ x^2 h(x) \in L_1(-\infty, +\infty), \ h_0(ux) \geq h_0(x)$ for all $u \in [0, 1]$ and for all $x \in (-\infty, +\infty), \ h_0 = h * h; \ *$ is the convolution operator$\}$.

Let us introduce into consideration the sequence of alternatives of the form ([1],[2]):

$$H_1 : \ f_1(x) = f_0(x) + \alpha_n \varphi\Big(\frac{x - \ell}{\gamma_n}\Big) + o(\alpha_n \gamma_n),$$

where $\alpha \downarrow 0$, $\gamma_n \downarrow 0$, $\ell$ is the fixed point of continuity $r(x)$ and $r(l) \neq 0$.

**Theorem.** *Let $K(x) \in H$ and $K_0(x) \in F$ ($F$ is the set of densities having bounded derivatives up to second order), $f_0(x) \in F$, $\varphi(x) \in F$. If $a_n = n^{\delta}$, $\alpha_n = n^{-\alpha}$, $\gamma_n = n^{-\beta}$ and also $\delta/2 = 1 - 2\alpha - \beta$, $\alpha + \beta > 1/2$, $2/9 < \delta \leq 1/2$, $\beta < 0,9\delta$, $\alpha < 2\delta$, then*

$$P_{H_1}\{U_n \geq \lambda_n(\alpha)\} \longrightarrow 1 - \Phi\Big(\varepsilon_\alpha - \frac{r(\ell)}{\sigma(f_0)} \int \varphi^2(u)\, du\Big),$$

*where*

$$\sigma(f_0) = 2 \int f_0^2(x) r^2(x)\, dx \int \Big( \int\limits_0^1 u^{\delta} K_0(u^{\delta} z)\, du \Big)^2 dz, \quad K_0 = K * K,$$

$$\lambda_n(\alpha) = \Delta(f_0) + \varepsilon_\alpha a_n^{-1/2} \sigma(f_0), \quad \Delta(f_0) = \gamma \int f_0(x) r(x)\, dx \int K^2(u)\, du, \quad \gamma = \frac{1}{1 + \delta}.$$

[1] Rosenblatt M. // Ann. Statist. — 1975. — **3**, 1–14.

[2] Nadaraya E. A. Nonparametric estimation of probability densities and regression curves. — Dordrecht: Kluwer Academic Publishers Group, 1989.