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On synchronization of nonlinear oscillations of two Berger plates coupled by internal subdomains

A problem of nonlinear oscillations of two plates elastically coupled by internal subdomains is considered. The plates, when undisturbed, occupy bounded domains Ω in different parallel planes and are assumed to be coupled by strictly internal open subdomain $\Omega_1 \subset \Omega$, whereas the part $\overline{\Omega_0} = \Omega \backslash \Omega_1$ are free from coupling. The edges of the plates are clamped. In the framework of Berger approach (see [1]) the problem can be described by the system of non-local partial differential equations

$$u_{itt} + \mu u_{it} + \Delta^2 u_i + \left(Q - \beta \|\nabla u_i\|^2_{L_2(\Omega)}\right) \Delta u_i + (-1)^i \gamma \chi(x)(u_2 - u_1) = p, \ i = 1, 2, \quad (1)$$

supplemented by clamped boundary conditions and initial data, where $u_i = u_i(x, t; \gamma), \ i = 1, 2$ denote the vertical with respect to equilibrium state displacement of the point $x \in \Omega$ of oscillating plates at time t for a given value γ. We study problem (1) under the assumption that the domain Ω_1 has smooth boundary $\partial\Omega_1$ so that the normal field ν is defined and continuous along $\partial\Omega_1$, and the parameters in system (1) are as follows: $\mu > 0, \ Q \in \mathbb{R}, \ \beta > 0, \ \gamma \geq 0$, and the function $\chi(x) \in C_0^\infty(\Omega)$ is positive for x from Ω_1 and equals zero for x from $\overline{\Omega_0}$.

We study the problem in terms of the dynamical system theory (see, e.g. [2]). The dynamical system generated in the space $H = [H_0^2(\Omega)]^2 \times [L_2(\Omega)]^2$ by (1) for each fixed γ is considered. The system possesses a compact global attractor A^γ. The attractor is upper semi-continuous with respect to γ, i.e.

$$\lim_{\gamma \to \gamma_0} \sup\{dist_H(a, A^{\gamma_0}), \ a \in A^\gamma\} = 0, \ \gamma_0 \geq 0. \quad (2)$$

Relation (2) holds for $\gamma_0 = \infty$ as well. In this case the set A^∞ belongs to the space $\tilde{H} = \{y = (y_1, y_2, y_3, y_4) \in H : \ y_1(x) = y_2(x), \ y_3(x) = y_4(x), \ x \in \overline{\Omega_1}\}$ and is the attractor of a dynamical system generated in $\tilde{H}$ by a suitable abstract problem. This implies the partial synchronization phenomenon in the sense that

$$\lim_{\gamma \to \infty} \lim_{t \to \infty} \left(\|u_1(x, t; \gamma) - u_2(x, t; \gamma)\|_{H^2(\Omega_1)} + \|u_{1t}(x, t; \gamma) - u_{2t}(x, t; \gamma)\|_{L_2(\Omega_1)}\right) = 0.$$

Similar results for several Berger plates coupled on the *whole* domain Ω are found in [3] and [4].

[1] Berger M. // J. Appl. Mech. — 1955. — N 22.

[2] Chueshov I.D. Introduction to the theory of infinite-dimensional dissipative systems. — Kharkov: Acta, 2002; see also http://www.emis.de/monographs/Chueshov/

[3] Naboka O. // Nonlinear Anal.: TMA — 2007. — N 67.

[4] NabokaO. // J. Math. Anal. — 2008. — N 341.