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On a Heyde characterization theorem for discrete Abelian groups

A lot of studies were devoted to characterizations of a Gaussian distribution on the real line. Specifically, in 1970 Heyde proved the following theorem.

Heyde theorem ([1, §13.4.1]). *Let ξ_j, $j = 1, 2, ..., n$, $n \geq 2$, be independent random variables. Let α_j, β_j be nonzero constants such that $\beta_i\alpha_i^{-1} \pm \beta_j\alpha_j^{-1} \neq 0$ for all $i \neq j$. If the conditional distribution of $L_2 = \beta_1\xi_1 + \cdots + \beta_n\xi_n$ given $L_1 = \alpha_1\xi_1 + \cdots + \alpha_n\xi_n$ is symmetric then all random variables ξ_j are Gaussian.*

Let X be a locally compact separable Abelian metric group, $\mathrm{Aut}(X)$ the set of topological automorphisms of X. Let ξ_j, $j = 1, 2, ..., n$, $n \geq 2$, be independent random variables with values in X and distributions μ_j. Consider the linear forms $L_1 = \alpha_1\xi_1 + \cdots + \alpha_n\xi_n$ and $L_2 = \beta_1\xi_1 + \cdots + \beta_n\xi_n$, where $\alpha_j, \beta_j \in \mathrm{Aut}(X)$ such that $\beta_i\alpha_i^{-1} \pm \beta_j\alpha_j^{-1} \in \mathrm{Aut}(X)$ for all $i \neq j$. Formulate the following problem.

Problem 1. Describe groups X for which the symmetry of the conditional distribution of the linear form L_2 given L_1 implies that all distributions μ_j are either Gaussian or belong to a class of distributions that can be considered as a natural analogue of the class of Gaussian distributions.

Problem 1 have not been solved, nevertheless it was studied in different important subclasses of the class of locally compact Abelian groups. In [2] Problem 1 was completely solved in the class of finite Abelian groups, and then in [3] it was solved in the class of countable discrete Abelian groups. For these classes of groups the class of idempotent distributions can be regarded as a natural analogue of the class of Gaussian distributions. In both cases a corresponding class of groups can be easily described. It consists of groups containing no elements of order two.

Formulate now the following general problem.

Problem 2. Let X be a locally compact separable Abelian metric group. Assume that the conditional distribution of the linear form L_2 given L_1 is symmetric. Describe possible distributions μ_j.

Problem 2 was solved in the class of finite Abelian groups in [4]. We solve Problem 2 in the class of countable discrete Abelian groups.

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[4] Mironyuk M.V., Fel'dman G.M. // Siberian Math. J. — 2005, — **46**, N 2.