Mogylova Viktoriia (National Technical University of Ukraine "KPI", Ukraine, Kyiv)
Novak Irina (Taras Shevchenko National University OF Kyiv, Ukraine, Kyiv)

Sufficient condition of oscillation for linear SDE of the second orders

We consider the stochastic differential equation Ito of the second order

$$\ddot{x} + (p(t) + q(t)\dot{W}(t))x = 0 \tag{1}$$

Here, $x \in R, t \geq 0$, $w(t)$ - is scalar process of Brownian motion defined on some probability space $(\Omega, \{F_t\}_{t\geq 0}, P)$; $p(t), \tilde{p}(t), q(t) \in C([0;\infty))$.

We will understand equation (1) as system

$$\begin{aligned} dx_1 &= x_2 dt, \\ dx_2 &= -p(t)x_1 dt - q(t)x_1 dW(t). \end{aligned} \tag{2}$$

Definition 1 *Assume $x(\tau(\omega), \omega) = 0$ with P1 then variable $\tau = \tau(\omega), \omega \in \Omega$ is zero of solution $x(t) = x(t, \omega)$ of equation (1).*

Definition 2 *Solution $x(t)$ of equation (1) have zero on interval I, if zero of solution of equation (1) exists and belongs I with P1.*

Definition 3 *Solution $x(t)$ of equation (1) is called oscillatory on I if $x(t)$ have two zeroes τ_1, τ_2 on I and hold $\tau_1 < \tau_2$ with P1.*

Lemma 1 *Quantity of zeroes of nontrivial solution of equation (1) are finish on any finish interval I with P1.*

Also we consider another stochastic differential equation Ito of the second order

$$\ddot{y} + (\tilde{p}(t) + q(t)\dot{w}(t))y = 0$$

and corresponding system

$$\begin{aligned} dy_1 &= y_2 dt, \\ dy_2 &= -\tilde{p}(t)y_1 dt - q(t)y_1 dW(t). \end{aligned} \tag{3}$$

Theorem 1 *If $\tilde{p}(t) \geq p(t)$ hold on I and τ_1, τ_2 are two consecutive zeroes of solution of equation (1) then any solution of equation (2) has one zero τ that satisfies properties $\tau_1 \leq \tau \leq \tau_2$ with P1.*

Theorem 2 *If τ_1, τ_2 are two consecutive zeroes of solution $x(t)$ of equation (1) and $\tilde{x}(t)$ linear-independence $x(t)$ hold then only one zero of $\tilde{x}(t)$ exists and satisfies properties $\tau_1 < \tau < \tau_2$ with P1.*

Investigation 1 *If one nontrivial solution of equation (1) has more than two zeroes on I, then all solutions of this equation are oscillatory.*

Definition 4 *Nontrivial solution or equation (1) is called oscillatory on $[0;\infty)$ with P1, if it has infinite set of zeroes on $[0;\infty)$.*

We denote τ_k – sequence of zeroes and $\varsigma_k = |\tau_{k+1} - \tau_k|$ – distance between consecutive zeroes.

We consider equation

$$\ddot{x} + (a^2 + q(t)\dot{W}(t))x = 0. \tag{4}$$

Theorem 3 *If condition*

$$\int\limits_0^\infty q^2(t)dt < \infty. \tag{5}$$

holds.

Then all nontrivial solutions $x(t)$ of equation (4) are oscillatory on $[0,\infty)$ and with P1

$$\varsigma_k \to \frac{\pi}{a},\ k \to \infty.$$

We denote

$$m = \inf_{t\geq 0} p(t), M = \sup_{t\geq 0} p(t).$$

Theorem 4 *If $0 < m < M < \infty$ and*

$$\int\limits_0^\infty q^2(t)dt < \infty. \tag{6}$$

hold. Also function $p(t)$ is not equal m and M. Then all solutions $x(t)$ of equation (1) are oscillatory on $[0,\infty)$.

And for all $\varepsilon > 0$: $\varepsilon < \frac{2}{\sqrt{M}}$ exists $T(\omega) > 0$: $\tau_n \geq T$ that for all nontrivial solutions $x(t)$ of equation (1) with P1 are performed

$$\frac{\pi}{\sqrt{M}} - \varepsilon \leq \tau_{n+1} - \tau_n \leq \frac{\pi}{\sqrt{m}} + \varepsilon$$