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## Certain Anti-holomorphic Submanifolds in a Locally Conformal Kaehler Manifold

In this talk, we define the $\mathcal{D}$-mean cuevature vector field $H_{\mathcal{D}}$ in an anti-holomorphic submanifold. Next, we consider special anti-holomorphic submanifolds which is called an almost contact anti-holomorphic submanifold, in a locally conformal Kaehler (an l.c.K.-) manifold. Then, we mainly prove the following theorems;

**Theorem A.** *In an almost contact anti-holomorphic submanifold in an l.c.K.-manifold, the $\mathcal{D}$-mean curvature vector field $H_{\mathcal{D}}$ satisfies*

$$\tilde{g}(\tilde{g}(\varphi V, \varphi U)H_{\mathcal{D}} - \frac{3}{2}\sigma(V,U)) - \frac{1}{2}\sigma(\varphi^2 V, U), \omega W)$$

$$= \tilde{g}(\tilde{g}(\varphi W, \varphi U)H_{\mathcal{D}} - \frac{3}{2}\sigma(W,U) - \frac{1}{2}\sigma(\varphi^2 V, U), \omega V)$$

*if and only if the morphisms $\varphi$ and $\omega$ satisfy*

$$\varphi[\varphi,\varphi](V,W) - \tilde{g}(\alpha_2^\sharp, \omega V)\varphi^2 W + \tilde{g}(\alpha_2^\sharp, \omega W)\varphi^2 V$$

$$+2\{\tilde{g}(\beta_1^\sharp, V)W - \tilde{g}(\beta_1^\sharp, W)V\} = 0$$

*for any $U, V, W \in TM$, where $[\varphi,\varphi]$ is the Nijenhuis tensor with respect to $\varphi$.*

**Theorem B.** *In a normal almost contact anti-holomorphic submanifold in an l.c.K.-manifold, we have*

$$\sigma(Y,X) = H_{\mathcal{D}}\tilde{g}(X,Y)$$

*for any $X, Y \in \mathcal{D}$. From this, we have*

$$\sigma(e_i, e_j) = \delta_{ji} H_{\mathcal{D}}$$

*for an orthonormal frame $\{e_1, ..., e_{2p}\}$ of $\mathcal{D}$, that is,*

$$\sigma(e_i,e_j) = \begin{pmatrix} H_{\mathcal{D}} & 0 & 0 & \mathbf{0} \\ 0 & H_{\mathcal{D}} & 0 & \\ 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 \\ \mathbf{0} & 0 & H_{\mathcal{D}} & 0 \\ & 0 & 0 & H_{\mathcal{D}} \end{pmatrix}.$$

*This means that the distribution $\mathcal{D}$ is totally umbilic in $\tilde{M}$.*

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