Schrödinger operators with pseudopotentials of the type
\(\alpha_1 \delta'(x) + \alpha_2 \delta'(x)\)

We consider one center Hamiltonians of the form

\[
H(\alpha_1, \alpha_2) = -\frac{d^2}{dx^2} + U(x) + \alpha_1 \delta'(x) + \alpha_2 \delta'(x)
\]  

(1)

here \(U\) is a smooth potential, \(\alpha_1\) and \(\alpha_2\) are coupling constants attached to the point source located at the origin, and \(\delta'(x)\) is the derivative of the Dirac function. The quantum mechanical particle thus moves under the influence of the potential \(U\) perturbed by a contact potential created by point sources of strength \(\alpha_1\) and \(\alpha_2\) located at the origin.

We approximate (1) by operators \(H_{\varepsilon, \gamma}(\alpha_1, \alpha_2, \Psi, \Phi)\) that are the closure of essentially selfadjoint ones

\[
H_{\varepsilon, \gamma}(\alpha_1, \alpha_2, \Psi, \Phi) = -\frac{d^2}{dx^2} + U(x) + \frac{\alpha_1}{\varepsilon^2} \Psi(\varepsilon^{-1} x) + \frac{\alpha_2}{\varepsilon^{2\gamma}} \Phi(\varepsilon^{-\gamma} x)
\]

with domain \(C_0^\infty(\mathbb{R})\). Here \(\Psi\) and \(\Phi\) are the shapes of the \(\delta'\)-like barrier that is to say \(\varepsilon^{-2} \Psi(\varepsilon^{-1} x)\) and \(\varepsilon^{-2\gamma} \Phi(\varepsilon^{-\gamma} x)\) tend to \(\delta'(x)\) in the sense of distributions, \(\gamma \geq 1\).

Applying asymptotic analysis we can construct the limit selfadjoint operators \(\mathcal{H} = \mathcal{H}_{\gamma}(\alpha_1, \alpha_2, \Psi, \Phi)\) corresponding to (1). The crucial point is that the asymptotic behavior of eigenvalues and eigensubspaces of \(\mathcal{H}\) strongly depends on the spectra of the Sturm-Liouville problems with an indefinite weight functions \(\Psi\) and \(\Phi\). Methods of operator theory in the Krein spaces are used here.