About a class of problem of pursuits in controlled distributed systems

The problem of pursuit of a following considered
\[
\sum_{\alpha=1}^{m} \frac{\partial}{\partial x_{\alpha}} \left( A^{(\alpha)}(x) \frac{\partial z}{\partial x_{\alpha}} \right) + \sum_{\alpha=1}^{m} B^{(\alpha)}(x) \frac{\partial z}{\partial x_{\alpha}} - C(x)z = -u + v, \quad z(x) = 0, \quad x \in \Gamma, \tag{1}
\]
where \( z = z(x) = (z^{1}(x), z^{2}(x), \ldots, z^{m_{0}}(x)) \), \( x = (x_{1}, x_{2}, \ldots, x_{p}) \in \omega \subset \mathbb{R}^{p} \), \( p \geq 1 \), \( \omega = \{0 \leq x_{\alpha} \leq 1, \ \alpha = 1, 2, \ldots, p\} \) with border \( \Gamma \), \( A^{(\alpha)}(x) \), \( B^{(\alpha)}(x) \) diagonal matrices \( m_{0} \times m_{0} \), which units is, accordingly \( a^{(m)}_{\alpha}(x), b^{(m)}_{\alpha}(x), C(x) \) — square matrix with units \( c^{(mm)}(x), a^{(m)}_{\alpha}(x), b^{(m)}(x), c^{(mm)}(x) \in \mathcal{C}^{1}(\omega) \); \( u \in \bar{P}, \ v \in \bar{Q} \) control functions, from a class \( L_{2}(\omega) \). In \( \bar{R}^{m_{0}} \) the terminal set \( \bar{M}_{1} \) is allocated. Game is considered ended, if \( z(\bar{x}) \in \varepsilon S + \bar{M}_{1} \), where \( S \) — an individual sphere from \( \bar{R}^{m_{0}} \). The differential problem for system (??) on a grid \( \bar{\omega} = \omega \cup \gamma \) with border \( \gamma \) looks like
\[
\frac{1}{h^{2}} \sum_{\alpha=1}^{m} \left[ A^{(\alpha)}_{i_{\alpha} - \frac{1}{2}, k} z_{i_{\alpha} - 1, k} - \left( A^{(\alpha)}_{i_{\alpha} - \frac{1}{2}, k} + A^{(\alpha)}_{i_{\alpha} + \frac{1}{2}, k} \right) z_{i_{\alpha}, k} + A^{(\alpha)}_{i_{\alpha} + \frac{1}{2}, k} z_{i_{\alpha} + 1, k} \right] + \\
+ \frac{1}{2h} \sum_{\alpha=1}^{m} B^{(\alpha)}_{i_{\alpha}, k} \left( z_{i_{\alpha} + 1, k} - z_{i_{\alpha} - 1, k} \right) - C_{i_{\alpha}, k} z_{i_{\alpha}, k} = -u_{i_{\alpha}, k} + v_{i_{\alpha}, k}, \tag{2}
\]
\[ z_{i_{\alpha}, k} = z_{i_{\alpha}, i_{2}, \ldots, i_{s}, k}, \quad i_{\alpha} = 0, r; \quad \alpha = 1, 2, \ldots, s; \quad k = 0, 1, \ldots, \theta. \]

To take into space \( \bar{R}^{m_{0}H} \) (here \( H \) — total number of knots belonging in \( \omega \) vector net functions, we will define in it scalar product and correcting the right member of equation (??) in frontier knots, we will write down differential problem (??) in the form of \( A\bar{z} = -\bar{u} + \bar{v}, \ \bar{z}, \bar{u}, \bar{v} \in \bar{R}^{m_{0}H} \). Applying Richardson’s [?] iterative method, we will receive the following game problem
\[
\bar{z}_{n+1} = (I - A)\bar{z}_{n} - \bar{u}_{n} + \bar{v}_{n}, \quad \bar{u}_{n} \in P, \quad \bar{v}_{n} \in Q. \tag{3}
\]

Let \( M = M_{0} + M_{1} \), where \( M_{0} \) — linear subspace \( \bar{R}^{m_{0}H} \), \( M_{1} \) — subset a subspace \( L \) — orthogonal addition \( M_{0} \) in \( \bar{R}^{m_{0}H} \). \( P \) matrix orthogonal designing from \( \bar{R}^{m_{0}H} \) on \( L \), \( P = \bar{P} \times \bar{P} \times \bar{P} \), \( Q = \bar{Q} \times \bar{Q} \times \bar{Q} \), \( M_{1} = \bar{M}_{1} \times \bar{M}_{1} \times \bar{M}_{1} \), \( 1 \leq \mu \leq H \) and \( W(0) = \{0\} \), \( W(n) = M_{1} + \sum_{\mu=0}^{\mu=n-1} \left[ \Pi(I - A)^{\mu} P \ast \Pi(I - A)^{\mu} Q \right] \), \( n = 1, 2, \ldots \).

**Theorem.** Suppose that \( N \) — least of those numbers \( n \), for each of which takes place inclusion \( \Pi(I - A)^{n}\bar{z}_{0} \in W(n) \). Then in game (??) from a point \( \bar{z}_{0} \) can be completed after \( N \) steps.