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About solving of general functional equations, having three subject variables, the set of invertible two-placed functions

The study of the functional equations, mentioned in the title, is reduced to the study of thirteen listed equations (see [1]). Two of them are distributive and Moufung functional equations and their solving is the well-known problem, which was considered by many authors. In particular, in [2] the parastrophic equivalence of three Moufung functional equations is proved and some subsets of their solution sets have been found. We solved the other eleven functional equations on the set of all invertible functions of an arbitrary set. For example, the following theorem is true.

Theorem. *A quintuple* $(f_1, \dots, f_5)$ *of invertible functions, defined on an arbitrary set* Q*, is a solution of the functional equation*

$$F_1(y; F_2(x; z)) = F_3(x; F_4(y; F_5(x; z))) \tag{1}$$

if and only if there exists a group $(Q; \cdot)$*, substitutions* α*,* β*,* γ*,* μ*,* ρ *and an invertible function* g*, being orthogonal to* $(\cdot)$*, such that* $f_1(x; y) = \alpha x \cdot \rho y$*,* $f_2(x; y) = \rho^{-1}(g^\ell(y; \gamma x) \cdot \gamma x)$*,* $f_3(x; y) = \beta y \cdot \gamma x$*,* $f_4(x; y) = \beta^{-1}(\alpha x \cdot \mu y)$*,* $f_5(x; y) = \mu^{-1} g^\ell(y; \gamma x)$*.*

Two functions f and g, defined on a set Q, are called *orthogonal*, if the system $f(x; y) = a$, $g(x; y) = b$ has a unique solution for all $a, b \in Q$.

Corollary 1. *A quintuple* $(f_1, \dots, f_5)$ *of invertible functions, which are topological on the topological line* $\mathbf{R}$ *with ordinary topology, is a solution of the functional equation* (1)*, if and only if there exist homeomorphisms* α*,* β*,* γ*,* μ*,* ν*,* ρ*,* φ *of the space* $\mathbf{R}$ *and an invertible topological function* g*, being orthogonal to the additive operation* $(+)$ *of the field* $\mathbf{R}$*, such that the following equalities are true:*

$$\begin{gathered} f_1(x; y) = \varphi(\alpha x + \rho y),\ f_2(x; y) = \rho^{-1}(g^\ell(\nu y; \gamma x) + \gamma x),\ f_3(x; y) = \varphi(\beta y + \gamma x), \\ f_4(x; y) = \beta^{-1}(\alpha x + \mu y), \quad f_5(x; y) = \mu^{-1} g^\ell(\nu y; \gamma x). \end{gathered} \tag{2}$$

Corollary 2. *A quintuple* $(f_1, \dots, f_5)$ *of invertible two-placed functions, defined on a set* Q*, where* $|Q| = p$ *is a prime number, is a solution of the functional equation* (1)*, if and only if there exist bijections* α*,* β*,* γ*,* μ*,* ν*,* ρ*,* φ^{-1} *between* Q *and* $\mathbf{Z}_p$*, an invertible function* g*, being orthogonal to* $(+)$*, such that the equalities* (2) *hold, where* $\mathbf{Z}_p$ *denotes the residue field modulo* p *and* $(+)$ *is the additive operation of the field* $\mathbf{Z}_p$*.*

[1] Sokhatsky F.M., Krainichuk H.V. About classification of the general functional equations, having three subject variables, on the set of invertible two-placed functions. (here)

[2] Belousov V.D. To Moufang functional equation // Voprosy quasigroup i lup. Kishinev: 1971, P. 11-19. (Russian)