1-D Schrödinger operators with local point interactions

Spectral properties of 1-D Schrödinger operators

\[ H_{X,\alpha} := -\frac{d^2}{dx^2} + \sum_{x_n \in X} \alpha_n \delta(x - x_n) \]

with local point interactions on a discrete set \( X = \{ x_n \}_{n=-\infty}^{\infty} \subset \mathbb{R} \) (\( \pm x_n \uparrow \pm \infty, \ n \to \pm \infty \)), are well studied when \( d^* := \inf_{n \in \mathbb{Z}} (x_n - x_{n-1}) > 0 \) (numerous results and a comprehensive list of references may be found in [1]; see also a survey of recent results given by Exner in [1, Appendix K]). In the case \( d^* = 0 \), it is only known that the operator \( H_{X,\alpha} \) may be symmetric with nontrivial deficiency indices (see [3]).

The main aim of our talk is the spectral analysis of the operators \( H_{X,\alpha} \) when \( d^* = 0 \). We show that the spectral properties of \( H_{X,\alpha} \) like self-adjointness, discreteness, and lower semiboundedness correlate with the corresponding spectral properties of certain classes of Jacobi matrices. Based on this connection, we obtain necessary and sufficient conditions for the operators \( H_{X,\alpha} \) to be self-adjoint, lower-semibounded, and discrete in the case \( d^* = 0 \).


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