Some generalized asymptotic properties of long-memory random fields with singular spectrum

Let $\xi(x)$, $x \in \mathbb{R}^n$ be a real, measurable, mean-square continuous, homogeneous isotropic Gaussian random field with $E\xi(x) = 0$, $E\xi^2(x) = 1$ and isotropic spectral function $\Phi(\lambda)$, $\lambda \geq 0$.

Let $\tilde{b}_{a}(r) = D\left[ \int_{\mathbb{R}^{n}} f_{n,r,a}(|t|)|t|^{n/2 - 1}\frac{J_{\nu}(r(\lambda - a))}{(r(\lambda - a))^{n/2}}J_{\nu-1}(|t|\lambda)d\lambda, \quad |t| \neq r \right]$, $\tilde{b}_{a}(r) = D\left[ \int_{\mathbb{R}^{n}} g_{n,r,a}(|t|)|t|^{n/2 - 1}\frac{J_{\nu}(r(\lambda + a))}{(r(\lambda + a))^{n/2}}J_{\nu-1}(|t|\lambda)d\lambda, \quad |t| \neq r \right]$, $J_{\nu}(z)$ – Bessel function of the first kind, $\nu > -\frac{1}{2}$.

Representations of weight functions $f_{n,r,a}(|t|)$, $g_{n,r,a}(|t|)$ by series are obtained and investigated.

Abelian and Tauberian theorems linking the local behavior of the spectral function $\Phi(x)$ in arbitrary point $x = a$ and the weighted integral functionals $\tilde{b}_{a}(r)$ and $\tilde{b}_{a}(r)$ of random fields are presented. The asymptotic behavior is described in terms of functions of the class OR. The difference of asymptotic behavior for functionals of the type $\frac{1}{r^\beta} \int_{\mathbb{R}^{n}} f_{n,r,a}(|t|)|t|^{\nu}d\lambda$ in the case of $a \neq 0$ is investigated.

The results generalize some properties of long-memory random fields. In a particular case $a = 0$ the classical results can be obtained easily.


