Borys Klykavka (Department of Probability Theory and Math. Statistics, Mathematics Faculty, Kyiv University)
Andriy Olenko (Department of Mathematics and Statistics, La Trobe University, Victoria, 3086, Australia)

Some generalized asymptotic properties of long-memory random fields with singular spectrum

Let $\xi(x),\ x \in R^n$ be a real, measurable, mean-square continuous, homogeneous isotropic Gaussian random field with $E\xi(x) = 0,\ E\xi^2(x) = 1$ and isotropic spectral function $\Phi(\lambda),\ \lambda \geq 0$.

Let

$$\tilde{b}^a(r) = D\left[\int_{R^n} f_{n,r,a}(|t|)\xi(t)dt\right],\quad \tilde{b}_a(r) = D\left[\int_{R^n} g_{n,r,a}(|t|)\xi(t)dt\right]$$

where

$$f_{n,r,a}(|t|) = \frac{1}{|t|^{\frac{n}{2}-1}}\int_0^\infty \lambda^{n/2}\frac{J_{\frac{n}{2}}(r(\lambda-a))}{(r(\lambda-a))^{n/2}}J_{\frac{n}{2}-1}(|t|\lambda)d\lambda,\quad |t| \neq r\,,$$

$$g_{n,r,a}(|t|) = \frac{1}{|t|^{\frac{n}{2}-1}}\int_0^\infty (\lambda+a)^{n/2}J_{\frac{n}{2}-1}(|t|(\lambda+a))\frac{J_{\frac{n}{2}}(r\lambda)}{(r\lambda)^{n/2}}d\lambda,\quad |t| \neq r\,,$$

$J_\nu(z)$ – Bessel function of the first kind, $\nu > -\frac{1}{2}$.

Representations of weight functions $f_{n,r,a}(|t|),\quad g_{n,r,a}(|t|)$ by series are obtained and investigated.

Abelian and Tauberian theorems linking the local behavior of the spectral function $\Phi(x)$ in arbitrary point $x = a$ and the weighted integral functionals $\tilde{b}^a(r)$ and $\tilde{b}_a(r)$ of random fields are presented. The asymptotic behavior is described in terms of functions of the class OR. The difference of asymptotic behavior for functionals of the type

$$\frac{1}{r^\beta}\int_{R^n} f_{n,r,a}(|t|)\xi(t)dt$$

in the case of $a \neq 0$ is investigated.

The results generalize some properties of long-memory random fields. In a particular case $a = 0$ the classical results can be obtained easily.

[1] Leonenko N.N. Limit theorems for random fields with singular spectrum. — Dordrecht/Boston/London: Kluwer Academic Publishers, 1999.

[2] Olenko A.Ya. Tauberian theorems for random fields with OR asymptotic II // Theory Probab. and Math. Statistics. —2006. — **74**.

[3] Klykavka B.M., Olenko A.Ya. Some properties of weight functions in Tauberian theorems I // Theory of Stochastic Processes. — 2006. — **12 (28)**, N 3-4.

[4] Olenko A.Ya. Some properties of weight functions in Tauberian theorems II // Theory of Stochastic Processes. —2007. — **13 (29)**, N 1-2.