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Eigenfunction Expansions for Relations Generated by Pair of Operator Differential Expressions

In a separable Hilbert space $\mathcal{H}$ we consider the operator differential equation

$$l\,[y] - \lambda m\,[y] = m\,[f]\,, \quad t \in \bar{\mathcal{I}},\ \mathcal{I} = (a,\ b) \subseteq R^1 \tag{1}$$

on finite or infinite intervals, where $l\,[y]$ and $m\,[y]$ are symmetric operator differential expressions of order r and s respectively, where $r + s > 0$, s is even. Let

$$\forall h_0, ..., h_{s/2} \in \mathcal{H}: \ \sum_{j=0}^{s/2} (\tilde{p}_j(t)h_j, h_j) + \Im \sum_{j=1}^{s/2} (\tilde{q}_j(t)h_{j-1}, h_j) \geq 0,\ t \in \bar{\mathcal{I}}, \tag{2}$$

where $\tilde{p}_j(t) = \tilde{p}_j^*(t)$, $\tilde{q}_j(t) \in B(\mathcal{H})$ are coefficients of the expression $m[y]$ that is presented in a formally symmetric form as in [1]. The leading coefficient of (1) has an inverse from $B\,(\mathcal{H})$ as $t \in \bar{\mathcal{I}}$, $\Im\lambda \neq 0$. In view of Lemma 2 from [1] condition (2) is equivalent to $W_\lambda(t) = \Im H_\lambda(t)/\Im\lambda \geq 0$, $t \in \bar{\mathcal{I}}$, $\Im\lambda \neq 0$ for the system $\frac{i}{2}\left((Q\,(t)\,x\,(t))' + Q^*\,(t)\,x'\,(t)\right) = H_\lambda\,(t)\,x\,(t)$, that is obtained from the homogeneous equation (1) by using quasi-derivatives. Therefore if (2) is valid, then this system contains the spectral parameter λ in Nevanlinna's manner. Also if $\exists\lambda_0$, $\Im\lambda_0 \neq 0 : W_{\lambda_0}(t_0) \geq 0$, then $\forall\lambda$, $\Im\lambda \neq 0$: $W_\lambda(t_0) \geq 0$.

In a special way we reduce the equation (1) to a symmetric first order system containing the spectral parameter either in a linear way (as $r > s$) or in a nonlinear way (as $r \leq s$). Using this reduction and the characteristic operator [2] of this system we construct a class of the generalized resolvents of the minimal relation corresponding to (1). These resolvents are integro-differential operators. From this the inversion formulas and Parseval's equality are obtained. For their proof we modify Strauss's method concerning the case of the generalized resovents as $s = 0$ and $m\,[y] \equiv y$ which are integral operators (but not integro-differential operators) depending on λ in more simple way comparing with the case $s > 0$. References to earlier expansions results for (1) can be found in [1].

Also the boundary value problems for the equation (1) with boundary conditions depending on the spectral parameter are considered. We show that for some boundary conditions, solutions of these problems are generated by a generalized resolvent if, in contrast to the case $s = 0$, the boundary conditions contain the derivatives of vector-function $f\,(t)$ that are taken on the ends of interval.

For $\dim\mathcal{H} < \infty$ we find the absolutely continuous part of the spectral matrix on the axis, when the coefficients of the equation (1) are periodic on the semi-axes and also we find the spectral matrix on the semi-axis, when the coefficients are periodic.

[1] Khrabustovskyi V. // Meth.Func.Anal.Topol. — 2009. — **15**, N 2.

[2] Khrabustovskyi V. // J.Math.Phys.Anal.Geom. — 2006. — **2**, N 2.