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Curvature forms and Curvature functions for 2-manifolds with boundary.

For 2-manifolds, possibly, with boundary the classical Gauss Bonnet formula asserts a relationship between the Euler characteristic of a manifold and its gaussian curvature and the geodesic curvature of the boundary. This is the only known obstruction on a given 2-form on a manifold to be the curvature form of some Riemmanian metric. Nevertheless, it imposes a constraint on the sign of a function for being the curvature function of a metric. The problem of prescribing curvature forms on closed 2-manifolds was solved by Wallach and Warner [4]. They showed that the Gauss Bonnet formula is a necessary and sufficient condition on a 2-form to be a curvature form. Later, the problem of prescribing curvature functions has been studied by some authers and completely solved for closed manifold by Kazadan and Warner [2]. They proved that any smooth function which satisfies Gauss Bonnet sign condition, is the gaussian curvature of some Riemmannian metric. In contrast with the case when manifolds have nonempty boundary no obstruction on 2-forms and functions arises. It turns out that any 2-form and smooth function can be realized as the curvature form and curvature function of a metric respectively, this is a suprizing phenomena.

Theorem 1. *Let M be a connected, compact and oriented 2-manifold with smooth boundary then any 2-form ω on M is the curvature form of some Riemannian metric g on M.*

Theorem 2. *Let M be a compact, connected and oriented 2-manifold with smooth boundary then any smooth function f is the gaussian curvature of some Riemmanian metric on M.*

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