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Some properties of Laguerre multiplier operators

Definition. *A real entire function f is said to belong to the class $\mathcal{L}-\mathcal{P}(-\infty;0)$ (the Laguerre-Pólya class), if it can be represented in the form*

$$f(z)=e^{-\beta z^2+\gamma z}\prod_{k=1}^{\infty}(1+\alpha_k z)e^{-\alpha_k z},$$

where γ is real, $\alpha_k>0, \beta\geq 0$, $\sum\alpha_k^2<\infty$.

Let $P(z)=\sum_{k=0}^{n}a_k z^k$ be a real polynomial having only positive zeros and $f\in\mathcal{L}-\mathcal{P}(-\infty;0)$. By the famous Pólya-Schur theorem the following polynomial $\Gamma_f(P)(z):=\sum_{k=0}^{n}a_k f(k)z^k$ have also only positive zeros. For $P(z)=C\prod_{k=1}^{n}(z-x_k),\ x_k>0$, we denote by $\delta(P)=\min_{1\leq k\leq n-1}\frac{x_{k+1}}{x_k}$

Theorem. *For every real polynomial P having only positive zeros and every $f\in\mathcal{L}-\mathcal{P}(-\infty;0)$ the following inequality holds $\delta(P)\leq\delta(\Gamma_f(P))$. Moreover, if $f(z)\neq e^{\gamma z}$ the inequality is strict.*

We will also discuss some other results of such type.