Some properties of Laguerre multiplier operators

**Definition.** A real entire function $f$ is said to belong to the class $\mathcal{L} - \mathcal{P}(-\infty; 0)$ (the Laguerre-Pólya class), if it can be represented in the form

$$f(z) = e^{-\beta z^2 + \gamma z} \prod_{k=1}^{\infty} (1 + \alpha_k z) e^{-\alpha_k z},$$

where $\gamma$ is real, $\alpha_k > 0$, $\beta \geq 0$, $\sum \alpha_k^2 < \infty$.

Let $P(z) = \sum_{k=0}^{n} a_k z^k$ be a real polynomial having only positive zeros and $f \in \mathcal{L} - \mathcal{P}(-\infty; 0)$. By the famous Pólya-Schur theorem the following polynomial $\Gamma_f(P)(z) := \sum_{k=0}^{n} a_k f(k) z^k$ have also only positive zeros. For $P(z) = C \prod_{k=1}^{n} (z - x_k)$, $x_k > 0$, we denote by $\delta(P) = \min_{1 \leq k \leq n-1} \frac{x_{k+1}}{x_k}$

**Theorem.** For every real polynomial $P$ having only positive zeros and every $f \in \mathcal{L} - \mathcal{P}(-\infty; 0)$ the following inequality holds $\delta(P) \leq \delta(\Gamma_f(P))$. Moreover, if $f(z) \neq e^{\gamma z}$ the inequality is strict.

We will also discuss some other results of such type.