Representations of finite $p$-groups over commutative rings

Let $K$ be an integral domain of characteristic zero, which is not a field. A finite group $G$ is called wild over the ring $K$, if the description of non-equivalent matrix $K$-representations of the group $G$ includes the problem of the classification up to similarity of pairs of $n \times n$-matrices over some field for an arbitrary natural $n$.

The problem of the wildness of a finite $p$-group over the ring $K$ have been solved [1–4], if $K$ is a complete discrete valuation ring or $K$ is the ring of formal power series in $m$ indeterminate with coefficients from complete discrete valuation ring.

We have obtain the next results.

**Theorem 1.** Let $G$ be a finite $p$-group of order $|G|$ and $K$ ba an integral domain of characteristic zero, $K^*$ be the multiplicative group of the ring $K$ and $p \notin K^*$. The group $G$ is wild over the ring $K$ if one of the following conditions holds:

1) $G$ is a non-cyclic $p$-group and $p \neq 2$;
2) $G$ is the cyclic $p$-group of order $|G| = p^r$ ($r > 2$, $p \neq 2$);
3) $G$ is a non-cyclic $2$-group of order $|G| > 4$;
4) $G$ is the cyclic $2$-group of order $|G| > 8$;
5) $G$ is a non-cyclic $2$-group of order 4 or the cyclic $2$-group of order $|G| > 4$ and $K$ is a local ring with residue class field of characteristic 2, $\text{Rad} K \neq 2K$ (Rad $K$ is the Jacobson radical of the ring $K$);
6) $G$ is the cyclic $p$-group of order $p^2$ and $K$ is a local factorial ring, which is not a discrete valuation ring.

It is received also a number of results about degrees of indecomposable matrix representations of finite $p$-groups over integral domains.