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Translation-invariant Operators and Abstract Difference Variable Trace

Let $\mathfrak{H}$ be a separable complex Hilbert Space, $\mathcal{L}(\mathfrak{H})$ be a space of bounded linear operators over the $\mathfrak{H}$. Let $A \in \mathcal{L}(\mathfrak{H})$, and B be a self-adjoint (in general – unbounded) operator in $\mathfrak{H}$. We say that A is *translation-invariant* relatively the B **if** A commutes with B. The main examples of translation-invariant operators are as follows

$$(A_{\mathcal{K}}x)\left(\vec{t}\right) = \int_{\mathbb{R}^d} \mathcal{K}(\vec{t}, \vec{s})x(\vec{s})d\vec{s}, \qquad \vec{t} \in \mathbb{R}^d,\ x \in \mathfrak{H}$$

in the space $\mathfrak{H} = L_2\left(\mathbb{R}^d\right)$, $d \in \mathbb{N}$, with the kernel $\mathcal{K}$, satisfying

$$\mathcal{K}(\vec{t}+h, \vec{s}+h) = \mathcal{K}(\vec{t}, \vec{s}), \qquad \vec{t}, \vec{s} \in \mathbb{R}^d,\ h \in \mathbb{R},$$

where $\vec{t}+h \equiv (t_1+h, \ldots, t_d+h)$, $\vec{t} = (t_1, \ldots, t_d) \in \mathbb{R}^d$. These operators are translation-invariant wth respect to the generator of the group $(U(\tau)x)\left(\vec{s}\right) = x\left(\vec{s}+\tau\right)$, $\vec{s} \in \mathbb{R}^d$, $\tau \in \mathbb{R}$, and, in general, they *haven't bounded trace* [1]. This is a reason that in [1] for Hamilton operators of superconductors it was introduced the concept of *the difference variable trace* of these operators.

In the present talk the notion of the *generalized projection trace* is introduced. We prove that the space of operators $A \in \mathcal{L}(\mathfrak{H})$ with bounded generalized projection trace is not complete. To define the Banach space we construct the expansion $\mathcal{L}^{+-}(\mathfrak{H})$ of the space $\mathcal{L}(\mathfrak{H})$, and consider the subspace of operators $A \in \mathcal{L}^{+-}(\mathfrak{H})$ with with bounded generalized projection trace. Note that in the general case $\mathcal{L}^{+-}(\mathfrak{H})$ can not be interpreted as space of operators (even unbounded) over the space $\mathfrak{H}$.

[1] Petrina D.Ya. *Mathematical Foundations of Quantum Statistical Mechanics. Continuous Systems.* Amsterdam: Kluwer, 1995., 624 p.