Andrii Goriunov (Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)

Formally self-adjoint quasi-differential operators

Let $[a,b]$ be a closed interval, $m \in \mathbf{N}$. We denote by $W_2^{[m]}$ the space of functions $y(x) \in L_2([a,b],\mathbf{C}) =: L_2$ such that following Shin-Zettl quasi-differential expressions exist a. e. on $[a,b]$: $D_0 y := y$, $D_k y := D(D_{k-1}y) + \sum\limits_{s=0}^{k-1} p_{k,s}(x) D_s y$, $D := -i\frac{d}{dx}$, $k = \overline{1,m}$, where $\overline{p_{k,s}(x)} = p_{m-s,m-k}(x) \in L_1([a,b],\mathbf{C})$, $s = \overline{0,k-1}$. They contain classical differential expressions and many others [3]. We consider in Hilbert space L_2 the minimal operator $L_{min} y := D_m y$, $Dom(L_{min}) := \{y \in W_2^{[m]} : D_k y(a) = D_k y(b) = 0,\ k = \overline{0,m-1}\}$. It is a closed densely defined symmetric operator in L_2 with deficiency index (m,m).

Theorem 1 ([1]). *Triplet* $(\boldsymbol{C}^m, \Gamma_1, \Gamma_2)$, *where* Γ_1, Γ_2 *are mappings from* $W_2^{[m]}$ *into* $\boldsymbol{C}^m$ *such that:*

$\Gamma_1 y := i\left(D_{2n-1}y(a), ..., D_n y(a), -D_{2n-1}y(b), ..., -D_n y(b)\right)$,

$\Gamma_2 y := \left(D_0 y(a), ..., D_{n-1}y(a), D_0 y(b), ..., D_{n-1}y(b)\right)$, *for* $m = 2n$, *and*

$\Gamma_1 y := i\left(D_{2n}y(a), ..., -D_{n+1}y(b), iD_n y(b) + D_n y(a)\right)$,

$\Gamma_2 y := \Big(D_0 y(a), ..., D_{n-1}y(b), \left(-\frac{1}{2} + i\right) D_n y(b) + \left(1 - \frac{1}{2}i\right) D_n y(a)\Big)$, *for* $m = 2n+1$,

is a space of boundary values for symmetric operator L_{min}.

Theorem together with results of [2, Ch.3, §1] imply

Theorem 2 ([1]). *Restriction of* L_{max} *on the set of functions* $y(x) \in W_2^{[m}$ *satisfying homogeneous boundary condition*

$$(K - I)\,\Gamma_1 y + i\,(K + I)\,\Gamma_2 y = 0, \tag{1}$$

where K *is a unitary operator in* $\boldsymbol{C}^m$, *is a self-adjoint extension* L_K *of* L_{min}. *Inversely, for every self-adjoint extension* $\widetilde{L}$ *such unitary operator* K *in* $\boldsymbol{C}^m$ *that* $\widetilde{L} = L_K$ *exists. This relation between unitary operators* K *and self-adjoint extensions is bijetive.*

Different characterization of self-adjoint extensions of quasi-differential operators, based on classical GKN theory, may be found in [3]. However, our approach has advantage of bijectivity of parametrization. Its another important advantage is that formula (1) describes all maximal dissipative extensions, if K is contraction in $\mathbf{C}^m$. Also in [1] we use this approach to describe all generalized resolvents of L_{min}.

These results were obtained together with V. A. Mikhailets [1].

[1] Goriunov A. S., Mikhailets V. A. // Reports of NAS of Ukraine — 2009. — N 4 and N 9.

[2] Gorbachuk M. L., Gorbachuk V. I. Boundary value problems for operator differential equations. — Mathematics and its Applications (Soviet Series), 48. Kluwer Academic Publishers Group, Dordrecht, 1991.

[3] W. N. Everitt, L. Markus. Boundary Value Problems and Symplectic Algebra for Ordinary Differential and Quasi-differential Operators. — AMS Bookstore, 1998.