Isaac A. García (Departament de Matemàtica, Universitat de Lleida, Spain.)
Héctor Giacomini (Laboratoire de Mathématiques et Physique Théorique, Faculté des Sciences et Techniques, Université de Tours, France)
Maite Grau (Departament de Matemàtica, Universitat de Lleida, Spain.)

The Cyclicity of some Monodromic Limit Sets in Analytic Planar Vector Fields

This work is concerned with planar real analytic differential systems with a smooth and non–flat inverse integrating factor defined in a neighborhood of a regular orbit. We show that the inverse integrating factor defines an ordinary differential equation for the transition map along the orbit. When the regular orbit is a limit cycle γ_0, we can determine its associated Poincaré return map in terms of the inverse integrating factor. In particular, we show that the multiplicity of γ_0 coincides with the vanishing multiplicity of an inverse integrating factor γ_0. We also apply this result to study the homoclinic loop bifurcation. We only consider homoclinic loops Γ whose critical point is a hyperbolic saddle s_0 and whose Poincaré return map is not the identity. A local analysis of the inverse integrating factor in a neighborhood of s_0 allows us to determine the cyclicity of this polycycle Γ in terms of the vanishing multiplicity of an inverse integrating factor over Γ. Our result also applies in the particular case in which the saddle s_0 of Γ is linearizable, that is, the case in which a bound for the cyclicity of this graphic cannot be determined through an algebraic method.

Finally, we also study the maximum number of limit cycles that can bifurcate from a focus singular point p_0 of an analytic, autonomous differential system in the real plane under an analytic perturbation. We consider p_0 being a focus singular point of the following three types: non-degenerate, degenerate without characteristic directions and nilpotent. In a neighborhood of p_0 the differential system can always be brought, by means of a change to (generalized) polar coordinates (r, θ), to an equation over a cylinder in which the singular point p_0 corresponds to a limit cycle γ_0. This equation over the cylinder always has an inverse integrating factor which is smooth and non–flat in r in a neighborhood of γ_0. The vanishing multiplicity of the inverse integrating factor over γ_0 determines the maximum number of limit cycles that bifurcate from the singular point p_0 in the non-degenerate case and a lower bound for the cyclicity otherwise.

[1] I.A. García, H. Giacomini and M. Grau, *The inverse integrating factor and the Poincaré map*, Trans. Amer. Math. Soc., to appear. `arXiv:0710.3238v1 [math.DS]`

[2] I.A. García, H. Giacomini and M. Grau, *Generalized Hopf bifurcation for planar vector fields via the inverse integrating factor*. `arXiv:0902.0681v1 [math.DS]`