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On behavior of solutions of degenerated nonlinear parabolic equations

The aim of this work is studing the behavior of solutions of initial boundary problem for degenerated nonlinear parabolic equation of the second order, conditions of existence and non-existence in whole by time solutions, is establish.

Let's consider the equation

$$\frac{\partial u}{\partial t} = \sum_{i,j=1}^{u} \frac{\partial}{\partial x_j} \left(\omega\left(x\right) \left| \frac{\partial u}{\partial x_i} \right|^{p-2} \frac{\partial u}{\partial x_i} \right) + f\left(x, t, u\right). \tag{1}$$

In bounded domain $\Omega \subset R^n$, $n \geq 2$ with nonsmooth boundary, namely the boundary $\partial\Omega$ contains the conic points with mortar of the corner $\omega \in (0, \pi)$. Besides the function f is measurable on whole arguments and not decrease by u. Let's consider the Dirichlet boundary condition

$$u = 0, x \in \partial\Omega \tag{2}$$

and the initial condition

$$\left. u \right|_{t=0} = \varphi\left(x\right) \tag{3}$$

in some domain Ω, where $\varphi\left(x\right)$ is a smooth function. Further we'll weak this condition.

Solution of problem (1) – (3) either exist in Π_0 or

$$\lim_{t \to T-0} \max_{\Omega} \omega\left(x\right) \left|u\left(x,t\right)\right| = +\infty \tag{4}$$

at some $T = const$.

Assuming that $\omega\left(x\right)$ is measurable non-negative function satisfying the conditions of Makenkhoupt's condition

Theorem. *Let* $f\left(x,t,u\right) \geq \alpha_0 \left|u\right|^{\sigma-1} u$ *at* $(x,t) \in \Omega$, $t > 0, u \geq 0$, *where* $\sigma = const > 1, \alpha_0 = const > 0$. *There exists* $k = const > 0$ *such that if* $u\left(x,0\right) \geq 0$, $\int\limits_{\Omega} u\left(x,0\right) u_0\left(x\right) dx \geq k$,

$$\lim_{t \to T-0} \max_{\Omega} \left(\omega\left(x\right) u_0\left(x\right) u\left(x,t\right)\right) = \infty,$$

where $T = const > 0$.