Modification of method broken Euler finding of the solution of semi-periodical boundary value problem for systems of nonlinear hyperbolic equations with mixed derivative

On $\Omega = [0, \omega] \times [0, T]$ nonlocal boundary value problem for the system of nonlinear hyperbolic equations

\[
\frac{\partial^2 u}{\partial x \partial t} = f(x, t, u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}), \quad (1)
\]

\[
u(x, 0) = u(x, T), \quad x \in [0, \omega], \quad (2)
\]

\[
u(0, t) = \varphi(t), \quad t \in [0, T], \quad (3)
\]

is considered. By

\[
v(x, t) = \frac{\partial u(x, t)}{\partial x}, \quad w(x, t) = \frac{\partial u(x, t)}{\partial t}
\]

denote new unknown functions and the problem (1) – (3) is reduced to an equivalent problem

\[
\frac{\partial v}{\partial t} = f(x, t, u(x, t), w(x, t), v(x, t)), \quad (4)
\]

\[
v(x, 0) = v(x, T), \quad x \in [0, \omega], \quad (5)
\]

\[
u(x, t) = \varphi(t) + \int_{0}^{x} v(\xi, t) d\xi, \quad w(x, t) = \dot{\varphi}(t) + \int_{0}^{x} w(\xi, t) d\xi. \quad (6)
\]

The segment $[0, \omega]$ is separable on $N$ parts with step $h > 0 : Nh = \omega$. Functions $v^{(i)}(t)$, $\dot{v}^{(i)}(t)$, $w^{(i)}(t)$, $\dot{w}^{(i)}(t)$ we will define equalities $v^{(0)}(t) = 0$, $\dot{v}^{(0)}(t) = 0$, $w^{(0)}(t) = \varphi(t)$, $\dot{w}^{(0)}(t) = \dot{\varphi}(t)$. Assuming known $v^{(i-1)}(t)$, $w^{(i-1)}(t)$, $i = 1, N$, function $v^{(i)}(t)$ we will discover as a solution of a periodic boundary value problem

\[
\frac{dv^{(i)}}{dt} = f((i - 1)h, t, u^{(i-1)}(t), w^{(i-1)}(t), v^{(i)}), \quad t \in [0, T], \quad v^{(i)}(0) = v^{(i)}(T). \quad (7)
\]

Functions $u^{(i)}(t)$, $w^{(i)}(t)$ we will define equalities

\[
u^{(i)} = \varphi(t) + h \sum_{j=0}^{i} v^{(j)}(t), \quad (8)
\]

\[
u^{(i)} = \dot{\varphi}(t) + h \sum_{j=0}^{i} \dot{v}^{(j)}(t), \quad (9)
\]

By means of functions $u^{(i)}(t)$, $w^{(i)}(t)$, $v^{(i)}(t)$, $i = 1, N$, we will construct continuous on $\Omega$ functions $u^{(0)}_{h}(x, t)$, $w^{(0)}_{h}(x, t)$, $v^{(0)}_{h}(x, t)$.

The sufficient conditions of existence of the solution of problem (1) – (3) and convergence to this solution of functions $u^{(0)}_{h}(x, t)$ are received by $h \to 0$. 

---

Dulat Dzhumabaev,  
Institute of Mathematics,  
050010, Pushkin Str.,125, Almaty, KAZAKHSTAN  
e-mail: anar@math.kz, dzhumabaev@list.ru