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Modification of method broken Euler finding of the solution of semi-periodical boundary value problem for systems of nonlinear hyperbolic equations with mixed derivative

On $\overline{\Omega} = [0,\omega] \times [0,T]$ nonlocal boundary value problem for the system of nonlinear hyperbolic equations

$$\frac{\partial^2 u}{\partial x \partial t} = f(x,t,u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x}), \tag{1}$$

$$u(x,0) = u(x,T), \qquad x \in [0,\omega], \tag{2}$$

$$u(0,t) = \varphi(t), \qquad t \in [0,T], \tag{3}$$

is considered. By

$$v(x,t) = \frac{\partial u(x,t)}{\partial x}, \qquad w(x,t) = \frac{\partial u(x,t)}{\partial t}$$

denote new unknown functions and the problem (1) – (3) is reduced to an equivalent problem

$$\frac{\partial v}{\partial t} = f(x,t,u(x,t),w(x,t),v(x,t)), \tag{4}$$

$$v(x,0) = v(x,T), \qquad x \in [0,\omega], \tag{5}$$

$$u(x,t) = \varphi(t) + \int\limits_0^x v(\xi,t)d\xi, \qquad w(x,t) = \dot{\varphi}(t) + \int\limits_0^x v_t(\xi,t)d\xi. \tag{6}$$

The segment $[0,\omega]$ is separable on N parts with step $h > 0 : Nh = \omega$. Functions $v^{(0)}(t)$, $\dot{v}^{(0)}(t)$, $w^{(0)}(t)$, $\dot{w}^{(0)}(t)$ we will define equalities $v^{(0)}(t) = 0$, $\dot{v}^{(0)}(t) = 0$, $w^{(0)}(t) = \varphi(t)$, $\dot{w}^{(0)}(t) = \dot{\varphi}(t)$. Assuming known $v^{(i-1)}(t)$, $u^{(i-1)}(t)$, $w^{(i-1)}(t)$, $i = \overline{1,N}$, function $v^{(i)}(t)$ we will discover as a solution of a periodic boundary value problem

$$\frac{dv^{(i)}}{dt} = f((i-1)h,t,u^{(i-1)}(t),w^{(i-1)}(t),v^{(i)}), \quad t \in [0,T], \qquad v^{(i)}(0) = v^{(i)}(T).$$

Functions $u^{(i)}(t)$, $w^{(i)}(t)$ we will define equalities

$$u^{(i)} = \varphi(t) + h\sum_{j=0}^{i} v^{(j)}(t), \qquad w^{(i)} = \dot{\varphi}(t) + h\sum_{j=0}^{i} \dot{v}^{(j)}(t), \qquad t \in [0,T], \qquad i = \overline{1,N}.$$

By means of functions $u^{(i)}(t)$, $w^{(i)}(t)$, $v^{(i)}(t)$, $i = \overline{1,N}$, we will construct continuous on $\overline{\Omega}$ functions $u_h^{(0)}(x,t)$, $w_h^{(0)}(x,t)$, $v_h^{(0)}(x,t)$.

The sufficient conditions of existence of the solution of problem (1) – (3) and convergence to this solution of functions $u_h^{(0)}(x,t)$ are received by $h \to 0$.