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Singularly perturbed normal operators

Let N be a normal operator with the domain $D(N)$ in the separable Hilbert space $\mathcal{H}$. The normal operator $\tilde{N}$ is called singularly perturbed with respect N iff the set $D \subset \{f \in D(\tilde{N}) \cap D(N), |\tilde{N}f = Nf\}$ is dense in $\mathcal{H}$. If $D(N) = D\dot{+}R$, where $R = N^{-1}(\mathcal{H} \ominus ND)$ is n-dimensional subspace of $\mathcal{H}$, then the operator $\tilde{N}$ is called singularly perturbed of rank n with respect to the operator N and we denote $\tilde{N} \in \mathcal{P}_s^n(\mathrm{N})$.

Definition. *The vector $r \in D(N)$, such that $e^{-i\theta}Nr - e^{i\theta}N^*r = 2i\xi r$ and $Nr \notin D(N)$, is called admissible for the operator N with characteristics θ, ξ: $-\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$, $\xi \in \mathbf{R}^1$.*

Theorem. *The set $\mathcal{P}_s^1(N)$ is not empty iff there exists the admissible vector r of the operator N.*

If r is admissible vector with characteristics θ, ξ, and τ is an arbitrarily real number then the operator $\tilde{N}_{r,\tau}$, that has the domain

$$D(\tilde{N}_{r,\tau}) = D\dot{+}\{e^{-i\theta}(\tau + i\xi)r + N^*r\}, \; D = N^{-1}(\mathcal{H} \ominus \{Nr\}) \tag{1}$$

and acts as follows

$$\tilde{N}_{r,\tau}(f_0 + e^{-i\theta}(\tau + i\xi)r + N^*r) = Nf_0 + e^{-i\theta}(\tau + i\xi)Nr, \tag{2}$$

belong to the set $\mathcal{P}_s^1(N)$.

Each operator $\tilde{N} \in \mathcal{P}_s^1(N)$ has the description by (1) and (2). Such description gives the bijection of the set $\mathcal{P}_s^1(N)$ and the set of couple $\{R, \tau\}$, where R is the one-dimensional subspace containing admissible vector and τ is the self adjoint operator in R.

The operator $\tilde{N}_{r,\tau}$ has bounded inverse one in $\mathcal{H}$ iff $\tau + i\xi \neq 0$, with this connection $\tilde{N}_{r,\tau}^{-1} = N^{-1} + e^{i\theta}(\tau + i\xi)^{-1}(\cdot, Nr)N^*r$, were we take into account $\|Nr\| = \|N^*r\| = 1$.

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[1] Dudkin M.E., Nizhnik L.P. Singularly perturbed normal operators // submitted to the Functional analysis and applications 2009.

[2] Dudkin M.E. Singularly perturbed normal operators of rank one and its applications. — K., 2008. — 38 p. (Prepr./NAS Ukraine. Institute of mathematics; 2008.3)