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Hyperalgebras

A *hyperalgebra* is a non-empty set H with one n-ary *hyperoperation* $f : H^n \to \mathcal{P}(H)$, where $\mathcal{P}(H)$ is a collection of all non-empty subsets of H. Such hyper algebra is called an *n-ary hypersemigroup* if

$$f(f(x_1,\ldots,x_n),x_{n+1},\ldots,x_{2n-1}) = f(x_1,\ldots,x_i,f(x_{i+1},\ldots,x_{i+n}),x_{i+n+1},\ldots,x_{2n-1})$$

holds for all $x_1,\ldots,x_{2n-1} \in H$ and $i = 1,\ldots,n-1$. An element e of H (if it exists) is a weak neutral element if $x \in f(e,\ldots,e,x,e,\ldots,e)$ for every $x \in H$. There are n-ary hypersemigroups without neutral elements and hypersemigroups with one, two and more neutral elements.

A hyperalgebra $(H,\circ)$, where $x\circ y = f(x,e,\ldots,e,y)$, is called a *binary retract* of (H,f) induced by e.

Theorem. *If an n-ary hypersemigroup (H,f) has a weak neutral element, then for any binary retract $(H,\circ)$ of (H,f) induced by a weak neutral element there exists a mapping $\varphi : H \to H$ such that $\varphi(x\circ y) \subseteq \varphi(x)\circ\varphi(y)$ and*

$$f(x_1,\ldots,x_n) \subseteq x_1 \circ \varphi(x_2) \circ \varphi(x_3) \circ \ldots \circ \varphi(x_{n-1}) \circ x_n.$$

We characterize these n-ary hypersemigroups in which the above inclusion can by replaced the equality.

Some classification (up to isomorphisms) of binary retracts of a given n-ary hypersemigroup will be presented.

[1] Davvaz B., Dudek W.A., Vougiouklis T. *A generalization of n-ary algebraic systems*// Commun. Algebra —2009. —**37**. —1246 – 1263.

[2] Davvaz B., Dudek W.A., Mirvakili S. *Neutral elements, fundamental relations and n-ary hypersemigroups*// International J. Algebra Computation —2009. —**19**. —567 – 583.