A Random Version of Filipov and Bogoliubov Theorems for
Hyperbolic Functional Differential Inclusions

Throughout the paper, $a, b$ are two positive real numbers; $Q_{a,b}$ is the rectangle $[0,a] \times [0,b]$ with the Lebesgue $\sigma$-algebra.

Denote by $(\Omega, U, P)$ a complete probability space, $C^* (Q_{a,b}, R^n)$ the space of all absolutely continuous (in a Carathéodory sense) maps from $Q_{a,b}$ into $R^n$, $ACC (\Omega \times Q_{a,b}, R^n)$ the space of all measurable maps $w : \Omega \rightarrow C^* (Q_{a,b}, R^n)$ . Let $\Psi : \Omega \times C^* (Q_{a,b}, R^n) \rightarrow cdL^1 (Q_{a,b}, R^n)$ be a set-value map with non-empty closed and decomposable values in $L^1 (Q_{a,b}, R^n)$.

**Theorem.** Assume that
1) the multifunction $\omega \rightarrow \Psi (\omega, z)$ is $(U, B [L^1])$ - measurable;
2) there exists a measurable map $k : \Omega \rightarrow R^+$ such that for every $\omega \in \Omega, u, v \in C^* (Q_{a,b}, R^n)$ and every $(x, y) \in Q_{a,b}$
\[
h_{L^1 (Q_{a,b}, R^n)} (\Psi (\omega, u), \Psi (\omega, v)) \leq \int_0^x \int_0^y k (\omega) \| u (s, r) - v (s, r) \| dsdr;
\]
3) for any measurable $w : \Omega \rightarrow C^* (Q_{a,b}, R^n)$ there exists a measurable map $\rho : \Omega \rightarrow L^1 (Q_{a,b}, R^+)$ such that for every $(x, y) \in Q_{a,b}$
\[
d_{L^1 (Q_{a,b}, R^n)} [w_{xy} (\omega), \Psi (\omega, w (\omega))] \leq \int_0^x \int_0^y \rho (\omega) (s, r) dsdr,
\]
\[
w (\omega) (x, 0) = \alpha (\omega) (x), \ w (\omega) (0, y) = \beta (\omega) (y), \ \alpha (\omega) (0) = \beta (\omega) (0),
\]
\[
\omega \in \Omega, \ \alpha : \Omega \rightarrow AC ([0, a], R^n), \ \beta : \Omega \rightarrow AC ([0, b], R^n).
\]

Then for every $\beta > 0$ , there exists a random solution $z : \Omega \rightarrow C^* (Q_{a,b}, R^n)$ of the problem
\[
z_{xy} (\omega) \in \Psi (\omega, z (\omega)), \ z (\omega) (x, 0) = \alpha (\omega) (x), \ z (\omega) (0, y) = \beta (\omega) (y),
\]
such that for every $(\omega, x, y) \in \Omega \times Q_{a,b}$
\[
\| z (\omega) (x, y) - w (\omega) (x, y) \| \leq \int_0^x \int_0^y e^{k (\omega) (y-s)} \rho (\omega) (s, r) dsdr + \beta e^{k (\omega) xy}.
\]

If in addition there exists
\[
\lim_{N \rightarrow \infty} \frac{1}{MN} \int_0^M \int_0^N \left\{ \int_\Omega \Psi (\omega, z) dP (\omega) \right\} dx dy = \Psi (z),
\]
then we can prove a basic Bogoliubov theorem of the method of averaging for inclusion
\[
z_{xy} \in \varepsilon^2 \Psi (\omega, z), \ (x, y) \in Q_{le^{-1}, le^{-1}}.
\]