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Problems of classifying associative or Lie algebras over a field of characteristic not 2 and finite metabelian groups are wild

This is joint work with Vladimir Sergeichuk, Genrich Belitskii, Ruvim Lipyanski, and Arkady Tsurkov.

We consider

(a) the problem of classifying symmetric bilinear mappings $U \times U \to V$, and

(b) the problem of classifying skew-symmetric bilinear mappings $U \times U \to V$,

in which U and V are vector spaces over a field F of characteristic different from 2 and V is three-dimensional.

We prove that the problems (a) and (b) contain the problem of classifying pairs of matrices over F up to similarity. Then we show that

(i) the problem of classifying local commutative associative algebras over F with zero cube radical contains (a),

(ii) the problem of classifying Lie algebras over F with central commutator subalgebra of dimension 3 contains (b), and

(iii) the problem of classifying finite p-groups of exponent p with central commutator subgroup of order p^3 contains the problem (a) over the field F_p with p elements.

Therefore, the problems (a), (b), and (i)–(iii) are wild.

Note that the wildness of (a), (b), (i), and (ii) was proved in [1] if the field F is algebraically closed. Our purpose is removing this restriction on F, which admits, in particular, to prove the wildness of (iii).

[1] G. Belitskii, R. Lipyanski, and V.V. Sergeichuk. Problems of classifying associative or Lie algebras and triples of symmetric or skew-symmetric matrices are wild. *Linear Algebra Appl.*, 407:249–262, 2005.