Projection Methods for Solution of Fundamental Equation of Risk Theory

Let $F(u)$ be the distribution function of claims $Y_j(=Y) > 0$ with expectation $EY_j = \mu$, $K(u)$ be the distribution of waiting time $T_j(=T) > 0$ with expectation $ET_j = 1/\alpha$, and $c > \alpha\mu$ be the gross premium rate, $j \in \mathbb{N}$. Random variables $Y_j$ and $T_j$ are supposed to be mutually independent. The non-ruin probability of an insurance company, $\varphi(u)$, with initial capital $u$ satisfies the Feller-Lundberg integral equation \[ [1] \]

\[ \varphi(u) - \int_0^\infty dK(v) \int_{0+cv}^{u+cv} \varphi(u + cv - z) dF(z) = 0, \quad u \geq 0, \] (1)

which is the equation of the Wiener-Hopf type. We are interested by the solution $\varphi(u)$ which is a monotone nondecreasing function of $u$, satisfying the condition

\[ \varphi(u) \nearrow 1 \quad \text{when} \quad u \to +\infty. \] (2)

Exact integration of the problem (1)-(2) presents difficulties and in the majority of cases may be done only by numerical methods. Using the results of the works \[ [2]-[4] \], the applicability of the projection methods to the solution of the problem (1)-(2) is justified. Number of illustrative examples are given.