Evgeney Cheremnikh (Lviv Polytechnic National University, Lviv, Ukraine)

Non-selfadjoint Friedrichs' model and trace formula

We consider in the space $H = L^2_\rho(0,\infty)$ the model $T = S + V$, $V = A^*B$ where $(S\varphi)(\tau) \equiv \tau\varphi(\tau)$, $\tau > 0$ and $A, B : H \to G$ are some integral operators which act from H in auxiliary Hilbert space G. This model contains some particular case of selfadjoint model considered in [1]. For non-selfadjoint model we have, in addition, spectral singularities $\sigma_j > 0$. If $h(\tau)$ is rational function, bounded on $[0,\infty)$, $h(\tau) \to 0$, $\tau \to \infty$ then there exists the function $\xi(\tau)$ and the number $\{c_{lj}\}$ such that

$$\mathrm{tr}[h(T) - h(S)] = \sum_{l,j} c_{lj} h^{(l)}(\sigma_j) + \int_0^\infty \xi(\tau) \frac{\mathrm{d}}{\mathrm{d}\tau}(\mathcal{R}h(\tau))\mathrm{d}\tau \tag{1}$$

where the function $\mathcal{R}h(\tau)$ is defined by spectral singularities and coincides with $h(\tau)$ if there is not spectral singularities.

We prove the following statement: let

$$f_N(\lambda) \sim \sum_{i=1}^\infty \frac{a_{i,N}}{\lambda^i}, \quad \lambda \to \infty \tag{2}$$

are a sequence of asymptotic decomposition such that

$$\left| f_N(\lambda) - \sum_{i=1}^\infty \frac{a_{i,N}}{\lambda^i} \right| \le C \frac{N^{l+1}}{|\lambda|^{l+1}}, \quad |\lambda| \ge N, \quad l = 1, 2, \ldots .$$

Suppose that $f_N(\lambda) \to f(\lambda)$, $a_{i,N}(\lambda) \to a_i(\lambda)$ quickly then every power of $1/N$ then we have "limit" asymptotic decomposition

$$f(\lambda) \sim \sum_{i=1}^\infty \frac{a_i}{\lambda^i}, \quad \lambda \to \infty \tag{3}$$

Later we discuss the application of passing (2)–(3) to analysis of the relation (1).

[1] Buslaev V. S. // Probl. Math. Phys. — 1970. — N.4, P.48–60 (russia).