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On elements of order 2 in groups of unitriangular matrices

Let k be a field of characteristic 2 and $k^* = k \setminus 0$. Let $E(n)$ denotes the identity $n \times n$ matrix. By the order of an $n \times n$ matrix A we mean as usual the smallest $r > 0$ such that $A^r = E(n)$.

The author studies the conjugacy classes of a group $\mathbf{UT}_{\boldsymbol{n}}\boldsymbol{(k)}$ (of upper unitriangular $n \times n$ matrices over k) consisting of matrices of order 2.

Put $[1, n] = \{1, 2, \dots, n\}$, $[1, n]^2 = [1, n] \times [1, n]$ and $[1, n]^2_< = \{(p, q) \in [1, n]^2 \,|\, p < q\}$. We denote by $I_{ij}(n)$ the $n \times n$ matrix $C = (c_{pq}), p, q = 1, \dots, n$, with $c_{ij} = 1$ and $c_{pq} = 0$ for any $(p, q) \neq (i, j)$.

Let P be a subset in $[1, n]^2_<$. An element $x = (i, j) \in [1, n]^2_<$ is said to be P-isolated if, for every $y = (p, q) \in P \setminus \{x\}$, the set $\{i, j\} \cap \{p, q\}$ is empty. Denote by $\mathcal{I}_n$ the set of all $P \subset [1, n]^2_<$, $P \neq \varnothing$, such that every element $x \in P$ is P-isolated.

Let $X \in \mathcal{I}_n$ and λ be a map from X to k^*; instead of $\lambda((i, j))$ one writes $\lambda(i, j)$. Denote by $M(X, \lambda)$ the following matrix (of order 2) from $\mathbf{UT}_n(k)$:

$$M(X, \lambda) = E(n) + \sum_{(i,j) \in X} \lambda(i, j) I_{ij}(n).$$

The following theorem describes the above mentioned conjugacy classes.

Theorem. 1) *Let K be a conjugacy class in $\mathbf{UT}_{\boldsymbol{n}}\boldsymbol{(k)}$ that consist of elements of order 2. Then there are X and λ such that $M(X, \lambda) \in K$.*

2) *Let K_1, K_2 be conjugacy classes in $\mathbf{UT}_{\boldsymbol{n}}\boldsymbol{(k)}$, which contain $M(X_1, \lambda_1)$ and $M(X_2, \lambda_2)$, respectively. If $(X_1, \lambda_1) \neq (X_2, \lambda_2)$ then $K_1 \neq K_2$.*

The author also computes the number of the indicated conjugacy classes.