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On weakly density for linearly ordered topological spaces

In the work it is proved that the weakly density of linearly ordered topological spaces is equal to its hereditary weakly density.

Let X be a set linearly ordered with the relation $<$ and containing at least two elements. For $a, b \in X$, satisfying the relation $a < b$, assume $(a, b) = \{x \in X : a < x < b\}$, $(a, \rightarrow) = \{x \in X : a < x\}$. Such sets will be named intervals in X. The family B of all intervals in the linearly ordered set X generates the base of topology in X. A linearly ordered space is the space, the topology of that is induced by some linear ordering [1].

For a cardinal function φ, $h\varphi$ denotes the cardinal function, the value of that on the space X is equal to $\sup \varphi(Y)$ where supreme is taken by the all subspaces Y of X, i.e. $h\varphi(X) = \sup \{\varphi(Y) : Y \subset X\}$.

A set $A \subset X$ is called everywhere dense in X if $[A] = X$. The density of the space X is defined as the least cardinal number of the form $|A|$ where A is an everywhere dense subset of X. This cardinal number is denoted as $d(X)$. If $d(X) \leq \aleph_0$, then we say that the space is separable [1].

We say that the weakly density [2] of a topological space X is equal to $\tau \geq \aleph_0$ if τ is the least cardinal number such that there exists in X a π-base decomposable on τ centered systems of open sets, i.e. $B = \cup \{B_\alpha : \alpha \in A\}$ is the π-base where B_α is a centered system of open sets for any $\alpha \in A$, $|A| = \tau$.

The weakly density of a topological space A is denoted as $wd(X)$. If $wd(X) = \aleph_0$, then X is called weakly separable [3].

Theorem 1. [4] Let X be a linearly ordered topological space. Then $d(X) = hd(X)$.

In the present report we prove the following statements.

Theorem 2. Let X be a linearly ordered topological space. Then $d(X) = wd(X)$.

Theorem 3. Let X be a linearly ordered topological space. Then $wd(X) = hwd(X)$.

Corollary. Let X be a linearly ordered topological space. Then $hd(X) = hwd(X)$.

References

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