About derivations in algebras of measurable operators

Let $M$ be an von Neumann algebra, let $LS(M)$ be the $*$-algebra of all locally measurable operators affiliated with to $M$ and let $S(M)$ be the $*$-algebra of all measurable operators affiliated with to $M$. Let $\tau$ be a semi-finite normal faithful trace on $M$ and $S(M, \tau)$ be the $*$-algebra of all $\tau$-measurable operators affiliated with to $M$ ([1]).

Let $A$ be an algebra. The linear operator $\delta$ on $A$ is called derivation if $\delta(ab) = \delta(a)b + a\delta(b)$ for any $a, b \in A$. The derivation $\delta$ is called inner if $\delta(x) = [d, x] = dx - xd$ when $d \in A$.

**Theorem 1.** Let $A$ be an absolutely solid $*$-subalgebra in $LS(M)$ (that is if $x \in LS(M)$ and $y \in A$ are such that $|x| \leq |y|$, then $x \in A$) and let $M \subseteq A$. And let $d \in LS(M)$ be such that $[d, x] = dx - xd \in A$ for any $x \in A$. Then derivation $\delta(x) = [d, x]$ is inner on $A$.

**Corollary.** All spatial derivations on algebras $S(M)$ and $S(M, \tau)$ are inner.

**Theorem 2.** Let $M$ be a properly infinite von Neumann algebra and let $A$ be a $*$-subalgebra in $LS(M)$, $M \subseteq A$. Then any derivation $\delta$ on $A$ is Z-linear (that is $\delta(z) = 0$ for any $z \in Z(A)$, when $Z(A)$ is the center of $A$).