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## Analogue of the Sylvester-Cayley formula for invariants of $n$-ary form

The number $\nu_{n,d}(k)$ of linearly independed homogeneous invariants of degree $k$ for the $n$-ary form of degree $d$ is calculated. The following formula holds

$$\nu_{n,d}(k) = \sum_{s\in\mathcal{W}} (-1)^{|s|} c_{n,d}\big(k, (\rho - s(\rho))^*\big),$$

where $\mathcal{W}$ is the Weyl group of the Lie algebra $sl_n$, $(-1)^{|s|}$ is the sign of the element $s \in \mathcal{W}$, $\rho = (1, 1, \ldots, 1)$ is half the sum of the positive roots of $\mathfrak{sl}_\mathbf{n}$, the weight $\lambda^*$ is the unique dominant weight on the orbit $\mathcal{W}(\lambda)$ and $c_{n,d}\big(k, (m_1, m_2, \ldots, m_{n-1})\big)$ is the number of non-negative integer solutions of the system of equations

$$\begin{cases} 2\,\omega_1(\alpha) + \omega_2(\alpha) + \cdots + \omega_{n-1}(\alpha) = d\,k - m_1, \\ \omega_1(\alpha) - \omega_2(\alpha) = m_2, \\ \ldots \\ \omega_{n-2}(\alpha) - \omega_{n-1}(\alpha) = m_{n-1}, \\ |\alpha| = k. \end{cases}$$

Here $I_{n,d} := \{\boldsymbol{i} = (i_1, i_2, \ldots, i_{n-1}) \in \mathbb{Z}_+^{n-1}, |\boldsymbol{i}| \leq d\}$, $|\boldsymbol{i}| := i_1 + \cdots + i_{n-1}$, $N = \binom{d+n-1}{d}$, $\alpha := (\alpha_{\boldsymbol{j}_1}, \alpha_{\boldsymbol{j}_2}, \ldots, \alpha_{\boldsymbol{j}_N})$, $\boldsymbol{j}_k \in I_{n,d}$, $\omega_r(\alpha) = \sum_{\boldsymbol{i}\in I_{n,d}} i_r \alpha_{\boldsymbol{i}}$,


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