On uniform boundedness of L^p - norms of canonical products in the unit disc

I.E. Chyzhykov, S.R. Skaskiv
Ivan Franko National University of L'viv
chyzhykov@yahoo.com, skaskiv@bigmir.net

For an analytic function $f(z)$, $z \in \mathbb{D}$, where $\mathbb{D}$ is the unit disc and $p \geqslant 1$ define

$$m_p(r, \ln|f|) = \left(\frac{1}{2\pi} \int\limits_0^{2\pi} \left| \ln|f(re^{i\theta})| \right|^p d\theta \right)^{1/p}$$

Growth of $m_p(r, \ln|f|)$ was studied by C.N. Linden in [1].
It is well known that the Dzrbashian-Naftalevich-Tsuji canonical product

$$P(z, \{z_k\}, s) = \prod_{k=1}^{\infty} E\left(\frac{1-|z_k|^2}{1-\bar{z}_k z}, s \right),$$

where $E(w, s) = (1-w)\exp\{w + w^2/2 + ... + w^s/s\}$, $s \in \mathbb{Z}_+$ is an analytic function with the zero sequence $\{z_k\}$, provided that $\sum_k (1-|z_k|)^{s+1} < +\infty$.

Let $S(\varphi, \delta) = \{\zeta = \rho e^{i\theta},\ 1-\delta \leqslant \rho < 1,\ \varphi - \pi\delta \leqslant \theta < \varphi + \pi\delta\}$.
Suppose that $\{z_k\}$ satisfies the following condition $\exists \gamma \in (0, s+1]$, such that

$$\sum_{z_k \in S(\varphi,\delta)} (1-|z_k|)^{s+1} = O(\delta^\gamma), \delta \downarrow 0. \qquad (1)$$

Theorem.
If (1) is true, then:

i) $m_p(r, \ln|P|) < L \dfrac{1}{(1-r)^{s-\gamma+1}} \log \dfrac{1}{1-r}$ *if* $0 < \gamma < s+1$;

ii) $m_p(r, \ln|P|) < L \log^2 \dfrac{1}{1-r}$ *if* $\gamma = s+1$.

1. Linden C.N., Integral logarithmic means for regular functions, Pacific J. of Math., 1989, 138, no.1, 119–127.