I. M. Savostyanova, Vit. V. Volchkov (Donetsk, Ukraine)

LOCAL ANALOGUES OF THE HEMISPHERICAL TRANSFORM

Generalizations of the class of functions having zero integrals over balls of a fixed radius are studied. A description of such functions in the form of series in special functions is obtained.

Let $\mathbf{S^2}$ be the standard unit sphere in $\mathbf{R^3}$ with the inner metric d, $B_R = \{\xi \in \mathbf{S^2} : d(o, \xi) < R\}$ be the open geodesic ball of radius R with center at the point $o = (0,0,1) \in \mathbf{S^2}$. Let r be a fixed number belonging to the interval $(0;\pi)$ and $r < R$. Denote by $\overline{B}_r$ the closure of B_r. Let $O(3)$ be the orthogonal group in $\mathbf{R^3}$. For fixed $M \in \mathbf{Z}_+$ we put

$$V_{r,M}(B_R) = \{f \in L_{loc}(B_R) : \int_{B_r} f(\tau\xi)(\xi_1 + i\xi_2)^M d\xi = 0 \ \forall \tau \in O(3) : \tau\overline{B}_r \subset B_R\},$$

where ξ_1, ξ_2, ξ_3 are the Cartesian coordinates of a point $\xi \in \mathbf{S^2}$ and $d\xi$ is the area element on $\mathbf{S^2}$. Also let

$$f_k(\theta) = \frac{1}{2\pi}\int_0^{2\pi} f(\sin\theta\sin\varphi, \sin\theta\cos\varphi, \cos\theta)e^{-ik\varphi}d\varphi, \quad k \in \mathbf{Z},$$

$N_k(r) = \{\nu > k : P_\nu^{-k}(\cos r) = 0\}$, where P_ν^{-k} is the Legendre function on (-1,1).

Theorem 1. *Let $f \in C^\infty(B_R)$, $R \le \pi$. Then $f \in V_{r,M}(B_R)$ if and only if for any $k \in \mathbf{Z}$ the decomposition*

$$f_k(\theta) = \sum_{\nu \in N_{M+1}(r)} c_\nu P_\nu^{-|k|}(\cos\theta) + (\sin\theta)^{|k|} \sum_{j=0}^{M-|k|-1} \gamma_j (\cos\theta)^j \qquad (1)$$

holds, where $0 \le \theta < R$, $c_\nu, \gamma_j \in \mathbf{C}$, $c_\nu = O(\nu^{-a})$ as $\nu \to +\infty$ for any $a > 0$ and the second sum in (1) is vanish for $|k| \ge M$.

A similar result on Euclidean space was established before by V.V. Volchkov in [1]. For other results in this direction, see [2-4].

1. *Volchkov V. V.* Mean value theorems for a class of polynomials // Sibirsk. Mat. Zh. – 1994. – 35, 4. – P. 737 – 745. English transl.: Siberian Math. J.– 1994. – 35. – P. 656 – 663.

2. *Volchkov V. V.* Integral geometry and convolution equations. – Dordrecht: Kluwer Academic Publishers, 2003. – 454 p.

3. *Volchkov V. V., Volchkov Vit. V.* Harmonic analysis of mean periodic functions on symmetric spaces and the Heisenberg group. – London: Springer, 2009. – 671 p.

4. *Volchkov V. V., Volchkov Vit. V.* Offbeat integral geometry on symmetric spaces. – Basel: Birkhäuser, 2013. – 592 p.