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AVERAGING ASYMPTOTICS OF SOLUTIONS TO THE NAVIER-STOKES EQUATIONS

Turbulent regimes are arisen under a small viscosity (or equivalently under a high Reynolds number) and are associated with rapidly oscillating fluid dynamics. Moreover, in mathematical and numerical modeling it is known that rapidly oscillation effects arise under computer simulations of solutions of Navier-Stokes equations with a vanishing viscosity. But reasons of the effects are not clear, since the effects may be turbulent regimes or the numerical simulations may be incorrect. Some averaging results in the direction will be presented in the report.

Average of Navier-Stokes equations with periodic rapidly oscillating initial data and the vanishing viscosity will be discussed. We give averaging (homogenized) equations whose solutions determine approximations of solutions of the equations under consideration and estimate the accuracy of the approximations. These approximations and estimates shed light on the following interesting property of the solutions of the equations. When the viscosity is not too small, the approximations contain no rapidly oscillating terms, and the equations under consideration asymptotically smooth the rapid oscillations of the data; thus, the equations are asymptotically parabolic. If the viscosity is very small, the approximations can contain rapidly oscillating terms with zero means, and the equations are hyperbolic.

As an example, we remark a precise result. Let ε be a small positive parameter and (u,p) be a weak solution of the initial-boundary value problem for nonstationary Navier-Stokes equations

$$\begin{gathered} u_t' - \nu\,\Delta\, u + u\cdot\nabla u + \nabla p = F_\varepsilon \quad \text{in} \quad \Omega\times(0,T), \\ \operatorname{div} u = 0 \quad \text{in} \quad \Omega\times(0,T), \\ u\big|_{t=0} = 0 \quad \text{in} \quad \Omega, \qquad u = 0 \quad \text{on} \quad \partial\Omega\times(0,T), \end{gathered} \tag{1}$$

where $F_\varepsilon = F(t,x,x/\varepsilon)$, $F(t,x,y) \in L^2(0,T;L^2(\Omega;L^\infty_{per}(Y)/\mathrm{R})^n)$, $\Omega \subset \mathrm{R}^n$ is a bounded domain with a smooth boundary, T is a given positive number, and $2 \le n \le 4$. Here, a subscript *per* means 1-periodicity with respect to $y \in \mathrm{R}^n$ and $Y = [0,1]^n$ is a periodicity cell. Thus, by definition $F(t,x,y)$ is 1-periodic in y, $\int_Y F(t,x,y)\,dy = 0$ for a. e. $(t,x) \in (0,T)\times\Omega$, and the restriction of $F(t,x,y)$ to Y is an element of $L^2(0,T;L^2(\Omega;L^\infty(Y))^n)$.

Theorem. *Let* $\nabla_x F \in L^1(0,T;L^2(\Omega;L^\infty_{per}(Y)/\mathrm{R})^{n\times n})$ *and* (u,p) *is a solution of problem (1). Then, there are positive* ε_0 *and* ν_0 *such that*

$$\|\,u\,\|^2_{L^\infty(\,0,T\,;\,L^2(\Omega)^n)} + \nu\,\|\,\nabla u\,\|^2_{L^2(\,0,T\,;\,L^2(\Omega)^{n\times n})} \le C\,(\,\varepsilon^2 + \varepsilon^2\,\nu^{-1}\,),$$

and

$$\|\,p\,\|_{W^{-1,\infty}(0,T;L^2(\Omega)/\mathrm{R})} \le C\,(\,\varepsilon + \varepsilon^2\,\nu^{-1-n/4}\,),$$

where C *is independent of* ε *and* ν *whenever* $0 < \varepsilon \le \varepsilon_0$ *and* $0 < \nu \le \nu_0$.

Asymptotic and homogenization methods are used for the consideration according to [1] and [2]. The results are applicable to some Kolmogorov flows.

1. Sandrakov G. V. *The influence of viscosity on oscillations in some linearized problems of hydrodynamics. Izvestiya: Math., 2007, 71, 97–148.*

2. Sandrakov G. V. *On some properties of solutions of Navier-Stokes equations with oscillating data. J. Math. Sciences, 2007, 143, 3377–3385.*